Atomic Energy Education Society – Distance Learning Programme

Class – **VIII** Subject – **Mathematics**

Chapter – **7**: **CUBES AND CUBE ROOTS**

 **Hand-out (Module 2/3)**

***Some Interesting Patterns***

1. **Adding consecutive odd numbers**

Observe the following pattern of sums of odd numbers:

 1 = 1 = 1 3

 3 + 5 = 8 = 2 3

 7 + 9 + 11 = 27 = 3 3

 13 + 15 + 17 + 19 = 64 = 4 3

 21 + 23 + 25 + 27 + 29 = 125 = 5 3

 ………………

 A perfect cube number is obtained when consecutive odd numbers are added.

* Let us express 10 3 as the sum of odd numbers using the pattern.

 91 + 93 + 95 + 97 + 99 + 101 + 103 + 105 + 107 + 109 = 1000 = 10 3

* Let us express 11 3 as the sum of odd numbers using the pattern.

 111+113+115+117+119+121+123+125+127+129+131 = 1331 = 113

1. **Another Pattern**

 23 – 13 = 1 + 2 × 1 × 3

 3 3 – 23 = 1 + 3 × 2 × 3

 4 3 – 33 = 1 + 4 × 3 × 3

* Let us find the value of 203 – 193 using the above pattern.
* 20 3 – 19 3 = 1 + 20 x 19 x 3
* Find the value of 513 – 503 using the above pattern.
* 513 – 503  = 1 + 51 x 50 x 3
1. **Yet another pattern**

13 = 1

 13 + 23 = 9 = (1 + 2)2

 13 + 23 + 33 = 36 = (1 + 2 + 3)2

 13 + 23 + 33 + 43 = 100 = (1 + 2 + 3 + 4)2

 13 + 23 + 33 + 43 + 53 = 225 = (1 + 2 + 3+ 4 + 5)2

13 + 23 + 33 + 43 + 53 + 63 = 441 = (1 + 2 + 3 + 4 + 5 + 6)2

 …………………….

13 + 23 + 33 + ……………. + n3 = (1 + 2 + 3 + …………..+ n)2

Here, the sum of the cubes of the first n natural numbers is equal to the square of the sum of the first n natural numbers.

***Cubes and their prime factors***

Consider the prime factorisation of 12

**12** = **2 x 2 x 3**

Now consider the prime factorisation of 123

123 = 1728 = 2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3 = 23 x 23 x 33

**123** = **2 3 x 2 3 x 3 3**

We find that *each prime factor of a number appears three times in the prime factorisation of its cubes.*

Consider the prime factorisation of 216.

216 = 2 x 2 x 2 x 3 x 3 x 3 = 23 x 33 = (2 x 3)3 = 63

216 is a perfect cube.

 Here we can say that *in the prime factorisation of any number, if each factor appears three times, then the number is a perfect cube*.

***Smallest multiple that is a perfect cube***

We can find the smallest natural number by which a number can be multiplied or divided so as to get a perfect cube.

* Is 68600 a perfect cube? If not, find the smallest number by which 68600 must be ***multiplied*** to get a perfect cube.

 The prime factorisation of 68600 = 2 × 2 × 2 × 5 × 5 × 7 × 7 × 7.

 In this factorisation, we find that there is no triplet of 5.

 So, 68600 is not a perfect cube.

 To make it a perfect cube we multiply it by 5.

 Thus, 68600 × 5 = 2 × 2 × 2 × 5 × 5 × 5 × 7 × 7 × 7

 = 343000, which is a perfect cube

 Thus the smallest number by which 68600 must be ***multiplied*** to get a
 perfect cube is **5.**

* Is 1188 a perfect cube? If not, by which smallest natural number should 1188 be ***divided*** so that the quotient is a perfect cube?

 The prime factorisation of 1188 = 2 × 2 × 3 × 3 × 3 × 11.

 The primes 2 and 11 do not appear in groups of three.

 So, 1188 is not a perfect cube.

 In the factorisation of 1188, the prime 2 appears only two times and the
 prime 11 appears once.

 So, if we divide 1188 by 2 × 2 × 11 = 44, then the prime factorisation of
 the quotient will not contain 2 and 11.

 Hence the smallest natural number by which 1188 should be ***divided*** to
 make it a perfect cube is **44**.

 And the resulting perfect cube is 1188 ÷ 44 = 27 (=33).

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