

Ray Optics Work Sheet 2 3

Solved Examples

1. Calculate the radius of curvature of an equi-concave lens of refractive index 1.5, when it is kept in a medium of refractive index 1.33, to have a power of -5D?

SOLUTION:

Given $n_2=1.5$ and $n_1=1.33$ and Power of lens = - 5D

Using $P = \frac{1}{f}$ we have

$$\Rightarrow f = \frac{1}{P} = \frac{1}{-5} = -0.2m = -20cm$$

Using lens maker formula

$$\frac{1}{f} = (n_{21} - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1 \right) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \Rightarrow \frac{1}{-20} = \left(\frac{1.5}{1.33} - 1 \right) \left[-\frac{1}{R} - \frac{1}{R} \right] = (0.1278) \left[-\frac{2}{R} \right] = -\frac{0.2556}{R}$$

$$\Rightarrow R = 20 \times 0.2446 = 5.112cm$$

2. A tank is filled with water to a height of 15 cm. The apparent depth of a coin lying at the bottom of the tank is measured by a microscope to be 11.25 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.6 up to the same height, by what distance would the microscope have to be moved to focus on the coin again?

Solution:

Real depth of the coin = $h_1 = 15$ cm Apparent depth of the coin = $h_2 = 11.25$ cm ;

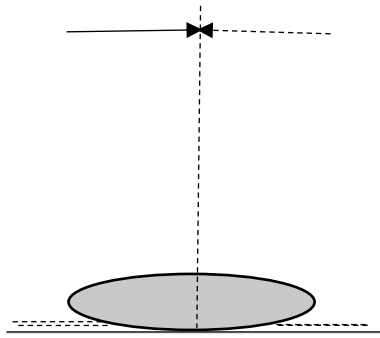
$$\text{Refractive index of water, } n = \frac{h_1}{h_2} = \frac{15}{11.25} = 1.33$$

When the water is replaced by a liquid of refractive index $n' = 1.6$

Refractive index of liquid is given by, $n' = \frac{h_1}{x}$ where x is the new apparent depth of the coin

$$\therefore x = \frac{h_1}{n'} = \frac{15}{1.6} = 9.375cm. \text{ To focus the coin again the microscope should be moved up by } 11.25 \text{ cm} - 9.375 \text{ cm} = 1.875 \text{ cm.}$$

3. A symmetric biconvex lens of radius of curvature R and made of glass of refractive index 1.5, is placed on a layer of liquid placed on top of a plane mirror as shown in the figure. An optical needle with its tip on the principal axis of the lens is moved along the axis until its real, inverted image coincides with the needle itself. The distance of the needle from the lens is measured to be x . On removing the liquid layer and repeating the experiment, the distance is found to be y . Obtain the expression for the refractive index of the liquid in terms of x and y .



Solution:

Let

- f = focal length liquid + glass lens
- f_1 = focal length of glass lens only
- f_2 = focal length of plano convex liquid lens

We have using combination of lenses

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} \Rightarrow \frac{1}{f_2} = \frac{1}{x} - \frac{1}{y} \Rightarrow \frac{1}{f_2} = \frac{y-x}{xy}$$

$$\Rightarrow f_2 = \frac{xy}{y-x} \text{ equation (1)}$$

Using lens maker's formula

$$\frac{1}{f} = (n_{21} - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\Rightarrow \frac{1}{f_1} = (n_{21} - 1) \left[\frac{1}{R} + \frac{1}{R} \right]$$

$$\Rightarrow \frac{1}{f_1} = (1.5 - 1) \left[\frac{2}{R} \right] \Rightarrow \frac{1}{y} = \frac{0.5 \times 2}{R} = \frac{1}{R} \Rightarrow R = y \text{ equation (2)}$$

For the liquid lens, we have

$$\frac{1}{f_2} = (n' - 1) \left[\frac{1}{R} - \frac{1}{\infty} \right] = (n' - 1) \left[\frac{1}{R} \right], \text{ } n' \text{ is refractive index of}$$

liquid

using equations(1) and(2)

$$\Rightarrow -\frac{y-x}{xy} = (n' - 1) \left[\frac{1}{y} \right]$$

Solving this we get, refractive index of liquid

$$n' = 2 - \frac{y}{x}$$

4. What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

Solution:

Focal length of the convex lens = 30 cm

Focal length of the concave lens = - 20 cm

Focal length of the system of lenses = f

Using the formula for the combination of lenses

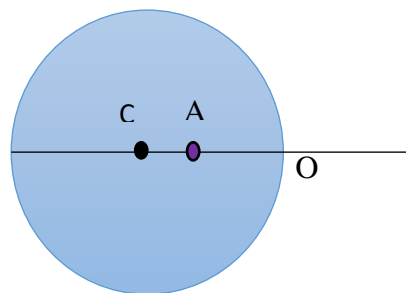
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{20} = -\frac{1}{60} \Rightarrow f = -60\text{cm}$$

The negative sign indicates that the system of lenses acts as a diverging lens

5. An air bubble is trapped inside a glass sphere of radius 15 cm at a distance of 6 cm from its centre. Where does it appear to an observer who is looking at it along the diameter from the side to which it is nearest? (Refractive index of glass=1.5)

Solution:



Let C be the centre of glass sphere, A be the air bubble and O be the observer

Distance CA= 6cm, Radius= 15 cm

Object distance = u= -(15-6)cm= - 9 cm

Refractive index of glass $n_1=1.5$ and Refractive index of air $n_2 =1$

Using the refraction of light at a spherical surface formula

$$\begin{aligned} \frac{n_2}{v} - \frac{n_1}{u} &= \frac{n_2 - n_1}{R} \\ \Rightarrow \frac{1}{v} - \frac{1.5}{-9} &= \frac{1-1.5}{-15} \\ \Rightarrow \frac{1}{v} &= \frac{0.5}{-15} + \frac{1.5}{-9} = -\left(\frac{27}{135}\right) \\ \Rightarrow v &= -5\text{cm} \end{aligned}$$

The air bubble appears to be seen at 5cm from the side of the observer.

Solve the following questions

1. Find the radius of curvature of the convex surface of a plane convex lens, whose focal length is 0.3m and the refractive index of the material of the lens is 1.5?
2. A convex lens has a focal length 0.2m and made of glass is immersed in water (refractive index=1.33) find the change in focal length of the lens?
3. An equi-convex lens of radius of curvature R is cut into two equal parts by a vertical plane, so it becomes a plano-convex lens. If f is the focal length of equi-convex lens, then what will be focal length of the plano-convex lens?
4. An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?
5. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, and (b) a concave lens of focal length 16 cm?
6. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20cm?
7. A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?
8. A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?
9. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)
10. An object is placed 30 cm in front of a plano-convex lens with its spherical surface of radius of curvature 20 cm. If the refractive index of the material of the lens is 1.5, find the position and nature of the image formed

Acknowledgement

The questions in this work sheet have been taken from NCERT text book and based on previous years CBSE Board Exam question papers