ATOMIC ENERGY EDUCATION SOCIETY, MUMBAI

CLASS: XII (MATHEMATICS)

CHAPTER - 09
TOPIC: DIFFERENTIAL EQUATIONS
HANDOUT: MODULE 2/3

Methods of Solving First Order, First Degree Differential Equations:-

(1) Differential equations with variables separable:

If a first order-first degree differential equation

$$\frac{dy}{dx} = f(x, y) \text{ ----- } (1)$$

can be expressed in the form

$$\frac{dy}{dx} = g(x).\, h(y),$$

then the differential equation (1) is said to be of variable separable type.

$$\Rightarrow \frac{dy}{h(y)} = g(x)dx$$

$$\Rightarrow \int \frac{dy}{h(y)} = \int g(x)dx, \quad h(y) \neq 0$$

Solution of the differential equation: H(y) = G(x) + C

Example:- (i) Find the general solution of the differential equation

$$x^4 \frac{dy}{dx} = -y^3$$

Separate the variables

$$\frac{dy}{y^3} = \frac{1}{x^4} dx$$

Integrate both the sides

$$\int \frac{dy}{y^3} = \int \frac{1}{x^4} dx$$

$$\int y^{-3} dy = \int x^{-4} dx \Rightarrow -\frac{1}{2\,y^2} = -\frac{1}{3\,x^3} + C.$$

(2) Homogeneous differential equations:

Homogeneous function:

A function F(x, y) is said to be homogeneous function of degree n if
F ($\lambda x, \lambda y) = \lambda^n$ F(x, y) for any nonzero constant λ.

Homogeneous DE:

A differential equation of the form $\frac{dy}{dx}$ = F (x, y) is said to be homogenous differential equation if F(x, y) is a homogenous function of degree zero.

Method of solving a homogeneous differential equation of the type:

$$\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$$

Make the substitution $y = v.\, x$

Diff. w. r. to x, we get

$$\frac{dy}{dx} = v.\,1 + x.\frac{dv}{dx}$$

Substituting the value of $\frac{dy}{dx}$ in the given differential equation, we get

$$v.\,1 + x.\frac{dv}{dx} = g(v)$$

$$x.\frac{dv}{dx} = g(v) - v$$

$$\frac{dv}{g(v) - v} = \frac{dx}{x}$$

Integrating both the sides, we get

$$\int \frac{dv}{g(v) - v} = \int \frac{dx}{x}$$

Which gives the general solution of the given differential equation when v is replaced by $\frac{y}{x}$.

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Example:- (i) Show that the differential equation

$x \operatorname{Cos}\left(\frac{y}{x}\right).\frac{dy}{dx} = y \operatorname{Cos}\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.

Ans: $$x \operatorname{Cos}\left(\frac{y}{x}\right).\frac{dy}{dx} = y \operatorname{Cos}\left(\frac{y}{x}\right) + x$$

$$\frac{dy}{dx} = \frac{y\,\mathrm{Cos}\left(\frac{y}{x}\right)+x}{x\,\mathrm{Cos}\left(\frac{y}{x}\right)} = F(x, y)$$

Replacing x by λx and y by $\lambda\, y$, we get

$$F(\lambda\, x, \lambda\, y) = \frac{\lambda\left(y\,\mathrm{Cos}\left(\frac{y}{x}\right)+x\right)}{\lambda\left(x\,\mathrm{Cos}\left(\frac{y}{x}\right)\right)} = \lambda^0[F(x,y)]$$

Thus, F(x, y) is a homogeneous function of degree zero.
Therefore, the given differential equation is a homogeneous differential equation.

$$\frac{dy}{dx} = \frac{\frac{y}{x}\mathrm{Cos}\left(\frac{y}{x}\right)+1}{\mathrm{Cos}\left(\frac{y}{x}\right)} \text{ ------- (i)}$$

Put $y = v.\, x$

Diff. w. r. to x, we get

$$\frac{dy}{dx} = v.\,1 + x.\,\frac{dv}{dx}$$

From (i)

$$v.\,1 + x.\,\frac{dv}{dx} = \frac{v\,\mathrm{Cos}\,v+1}{\mathrm{Cos}\,v}$$

Apply variable separable method in v & x

$$x.\,\frac{dv}{dx} = \frac{v\,\mathrm{Cos}\,v+1}{\mathrm{Cos}\,v} - v$$

$$= \frac{v\,\mathrm{Cos}\,v+1-v\,Cos\,v}{\mathrm{Cos}\,v}$$

$$x.\,\frac{dv}{dx} = \frac{1}{\mathrm{Cos}\,v}$$

$$\mathrm{Cos}\,v\,dv = \frac{dx}{x}$$

$$\int \mathrm{Cos}\,v\,dv = \int \frac{dx}{x}$$

$$\mathrm{Sin}\,v = \log|x| + C$$

$$\mathrm{Sin}\left(\frac{y}{x}\right) = \log|x| + C$$

Which is the general solution of the differential equation (1).