

ATOMIC ENERGY CENTRAL SCHOOL INDORE

CHAPTER : LAWS OF MOTION

CLASS : XI

SUBJECT : PHYSICS

Module : 3/3

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CIRCULAR MOTION

- Acceleration of a body moving in a circle of radius R with uniform speed v is v^2/R directed towards the centre.

According to the second law, the force f providing this acceleration is :

$$f = mv^2/R$$

where m is the mass of the body. This force directed towards the centre is called the centripetal force.

For a stone rotated in a circle by a string, the centripetal force is provided by the tension in the string.

The centripetal force for motion of a planet around the sun is the gravitational force on the planet due to the sun.

Motion on level curved road

- Three forces act on the car.
- (i) The weight of the car, mg (ii) Normal reaction, N
- (iii) Frictional force, f
- As there is no acceleration in the vertical direction

$$N - mg = 0$$

$$N = mg$$

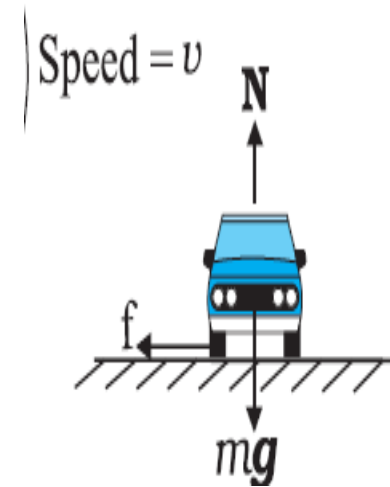
- The centripetal force required for circular motion is along the surface of the road, and is provided by the component of the contact force between road and the car tyres along the surface. This by definition is the frictional force. Note that it is the static friction that provides the centripetal acceleration. Static friction opposes the impending motion of the car moving away from the circle.

$$F = \mu N \geq mv^2/R$$

$$v^2 \leq \mu RN/m = \mu R g \quad [N = mg]$$

which is independent of the mass of the car. This shows that for a given value of μ and R , there is a maximum speed of circular motion of the car possible, namely

$$V_{\max} = \sqrt{\mu R g}$$



Motion of car on banked Road

- We can reduce the contribution of friction to the circular motion of the car if the road is banked. Since there is no acceleration along the vertical direction, the net force along this direction must be zero. Hence, $N \cos \vartheta = mg + f \sin \vartheta$
- The centripetal force is provided by the horizontal components of N and f .

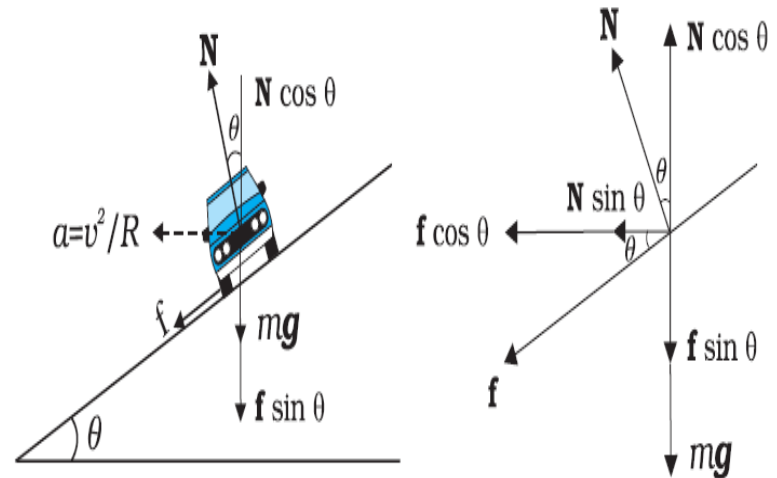
$$N \sin \vartheta + f \cos \vartheta = mv^2/R$$

Thus to obtain v_{\max} we put

$$f = \mu N .$$

Then Eqs. become

$$N \cos \vartheta = mg + \mu N \sin \vartheta$$



Motion of car on banked Road

- $N \sin \vartheta + \mu N \cos \vartheta = mv^2/R$ -----(1)

We obtain $N = mg / (\cos \theta - \mu \sin \theta)$

- Substituting value of N in Eq. 1, we get

$$Mg((\sin \theta + \mu \cos \theta) / (\cos \theta - \mu \sin \theta)) = m v_{\max}^2 / R$$

- $v_{\max} = \sqrt{(Rg(\mu + \tan \theta) / (1 - \mu \tan \theta))}$

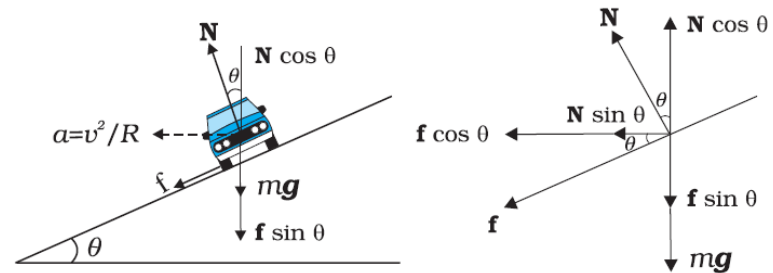
maximum possible speed of a car on a banked road is greater than that on a flat road.

For $\mu = 0$

$$v_0 = \sqrt{Rg \tan \theta}$$

At this speed, frictional force is not needed at all to provide the necessary centripetal force.

Driving at this speed on a banked road will cause little wear and tear of the tyres.



Example : Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the co-efficient of static friction between the box and the train's floor is 0.15.

Answer : Since the acceleration of the box is due to the static friction,

$$ma = fs \leq \mu_s N = \mu_s m g$$

$$\text{i.e. } a \leq \mu_s g$$

$$\begin{aligned} \therefore a_{\max} &= \mu_s g = 0.15 \times 10 \text{ ms}^{-2} \\ &= 1.5 \text{ m s}^{-2} \end{aligned}$$

Example: A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn ?

- **Answer :** On an unbanked road, frictional force alone can provide the centripetal force needed to keep the cyclist moving on a circular turn without slipping. If the speed is too large, or if the turn is too sharp (i.e. of too small a radius) or both, the frictional force is not sufficient to provide the necessary centripetal force, and the cyclist slips. The condition for the cyclist not to slip is given :

$$v^2 \leq \mu_s R g$$

Now, $R = 3 \text{ m}$, $g = 9.8 \text{ m s}^{-2}$, $\mu_s = 0.1$. That is,

$$v^2 = \mu_s R g = 2.94 \text{ m}^2 \text{ s}^{-2}.$$

$$v = 18 \text{ km/h} = 5 \text{ m s}^{-1}; \quad \text{i.e., } v^2 = 25 \text{ m}^2 \text{ s}^{-2}.$$

The condition is not obeyed. The cyclist will slip while taking the circular turn.



• Thank you