CLASS - XI

Chapter – 9

SEQUENCES AND SERIES

MODULE – 3 of 3

Distance Learning Programme: An initiative by AEES, Mumbai

7. Geometric Progression (G . P.)

Sequence of numbers such that the quotient of any two successive members of the sequence is a constant

Some Examples of G.P

- 1, 1, 1, 1, 1, 1, 1, 1, 1…..
- 2, 4, 8, 16, 32, 64, 128
- 243, 81, 27, 9, 3, 1, 1/3, 1/9
- 1, -3, 9, -27, 81, -243, 729

Let $a_1, a_2, a_3, \ldots, a_n, \ldots$ is *geometric sequence* or *geometric progression.*

Here a_1 is the *first term (a)* and the constant quotient between consecutive terms is called the *common ratio (r)* of the G.P.

So, if first term is a and common ratio is r then

G.P $\rightarrow a, ar, ar^2, ar^3, ar^4, \ldots\ldots\ldots\ldots$

Here in G.P.

1st term → a	*(in 1st term r^0)*
2nd term → ar	*(in 2nd term r^1)*
3rd term → ar^2	*(in 3rd term r^2)*
4th term → ar^3	*(in 4th term r^3)*
5th term → ar^4	*(in 5th term r^4)*
6th term → ar^5	*(in 6th term r^5)*
7th term → ar^6	*(in 7th term r^6)*
---------------------	-----------------
---------------------	-----------------
15th term → ar^{14}	*(in 15th term r^{14})*
---------------------	-----------------
---------------------	-----------------
nth term → ar^{n-1}	*(in nth term r^{n-1})*

Hence,
The General term of a G.P is

$$a_n = ar^{n-1}$$

Sum to n terms of a G .P.

Let $a_1, a_2, a_3, \ldots, a_n, \ldots$ is *geometric sequence* or *geometric progression.*
If S_n be the sum of first n terms of the G.P

$$S_n = a_1 + a_2 + a_3 + \ldots + a_n$$
$$S_n = a + ar + ar^2 + ar^3 \ldots + ar^{n-1} \qquad \text{-------} \ (1)$$

If $r = 1$

$$S_n = a + a + a + a + a + \ldots\ldots n \text{ times}$$
$$S_n = na$$

If $r \neq 1$

Multiply r both sides in equation (1)

$$rS_n = r(a + ar + ar^2 + ar^3 \ldots + ar^{n-1})$$
$$rS_n = ar + ar^2 + ar^3 \ldots + ar^{n-1} + ar^n \qquad \text{-------} \ (2)$$

Now from equation (1) – equation (2)

$S_n - rS_n = (a + ar + ar^2 + ar^3 \ldots + ar^{n-1}) - (ar + ar^2 + ar^3 \ldots + ar^{n-1} + ar^n)$

$(1 - r)\, S_n = a + (ar + ar^2 + ar^3 \ldots + ar^{n-1}) - (ar + ar^2 + ar^3 \ldots + ar^{n-1}) - ar^n$

$(1 - r)\, S_n = a - ar^n$

$$S_n = \frac{a\,(1 - r^n)}{1 - r}$$

Some Examples

Example-1

Find the 10th and *n*th terms of the G.P. 5, 25, 125,… .

Solution

Here $a = 5$ and $r = 25/5 = 5$.

Thus, $a_{10} = 5(5)^{10-1} = 5(5)^9 = 5^{10}$

and $a_n = ar^{n-1} = 5(5)^{n-1} = 5^n$.

Example-2

Which term of the G.P., 2, 8, 32, ... up to n terms is 131072?

Solution

Let 131072 be the nth term of the given G.P.

Here $a = 2$ and $r = 8/2 = 4$.

Therefore

$131072 = a_n = 2(4)^{n-1}$

$\frac{131072}{2} = 4^{n-1}$

$65536 = 4^{n-1}$

$4^8 = 4^{n-1}$

$n - 1 = 8$,

i.e., $n = 9$.

Hence, 131072 is the 9th term of the G.P.

Example-3

Find the sum of first 5 terms of the geometric series 1 + 3 + 9 + ……

Solution

Here $a = 1$ and $r = 3/1 = 3$

As $S_n = \frac{a\,(r^n - 1)}{r-1}$

So $S_5 = \frac{1\left(3^5 - 1\right)}{3-1} = \frac{243-1}{2} = \frac{242}{2} = 121$

8. Geometric Mean (G .M.)

- **One Arithmetic Mean**

  Given two numbers a and b. We can insert a number G between them so that a, G, b is an G.P. Such a number G is called the *geometric mean* (G.M.) of the numbers a and b.

As a, G, b is in G.P

So

$$\frac{G}{a} = \frac{b}{G}$$

$$G^2 = ab$$

$$G = \sqrt{ab}$$

The geometric mean of two positive numbers a and b is the number $\sqrt{ab}$.

Example

Find the geometric mean of 2 and 8.

Solution

Geometric mean G = $\sqrt{2 \times 8} = \sqrt{16} = 4$

➢ *n* Geometric Means

Given two numbers a and b. We can insert n numbers G_1, G_2, G_3, ……, G_n between them so that a, G_1, G_2, G_3, ……, G_n, b is an G.P.

Here,

1st term is a and $(n+2)th$ term is b.

$$a_{n+2} = a.r^{n+2-1}$$

$$b = a.r^{n+1}$$

$$\frac{b}{a} = r^{n+1}$$

$$r = \left(\frac{b}{a}\right)^{1/n+1}$$

So $G_1 = a.r$

$G_2 = a.r^2$

$G_3 = a.r^3$

$G_n = a.r^n$

Example

Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

Solution

Let G_1, G_2, G_3 be three numbers between 1 and 256 such that 1, G_1, G_2,G_3, 256 is a G.P.

Here $256 = 1 \,.\, r^4 \Rightarrow r = \pm 4$ (Taking real roots only)

Hence, for $r = 4$, $G_1 = ar = 4$, $G_2 = ar^2 = 16$, $G_3 = ar^3 = 64$

Similarly, for $r = -4$, numbers are -4,16 and -64.

9. Relationship Between A.M. and G.M.

Let A and G be A.M. and G.M. of two given positive real numbers a and b, respectively.

Then $A = \frac{a+b}{2}$ and $G = \sqrt{ab}$

Thus, we have $A - G = \frac{a+b}{2} - \sqrt{ab}$

$$A - G = \frac{a + b - 2\sqrt{ab}}{2}$$

$$A - G = \frac{\sqrt{a}^2 + \sqrt{b}^2 - 2\sqrt{ab}}{2}$$

$$A - G = \frac{\left(\sqrt{a} - \sqrt{b}\right)^2}{2} \geq 0$$

Hence,

$$A \geq G$$

****