STRAIGHT LINES

MODULE 2/3

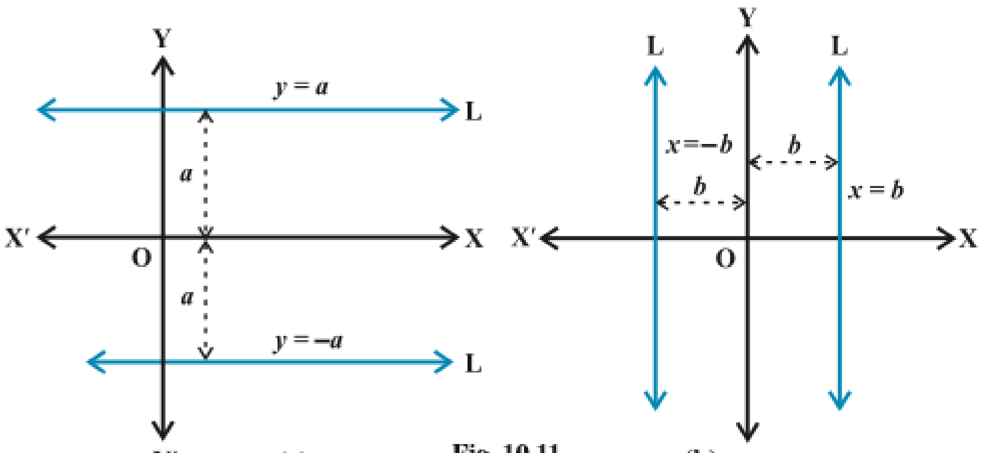
In this module we will study about

- Various form of equation of straight lines
  - Horizontal and vertical lines
  - Point - slope form
  - Slope - intercept form
  - Two point form
  - Slope-intercept form
  - Intercept form
  - Normal form
- Some example problems
- Problems for practice

Various forms of Straight Lines

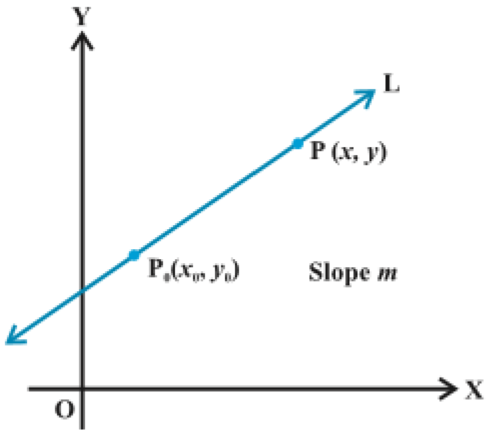
1.Horizontal and Vertical Lines :

If a horizontal line L is at a distance a from the x-axis, then the ordinate (y-coordinate) of every point lying on the line is either a or –a. So the equation of the line parallel to x-axis is y = a or y = - a. The choice of sign will depend upon the position of the line is above or below the x-axis. [Refer figure (a)]
Simillarly, equation of the vertical line at a distance b from y-axis is either
x = b or x = -b[Refer figure (b)]



(a) Fig 10.11 (b)

2.Point-slope form



Suppose $P_o(x_o,y_o)$is a fixed point on the line L. Let P(x,y) any arbitrary point on the line.

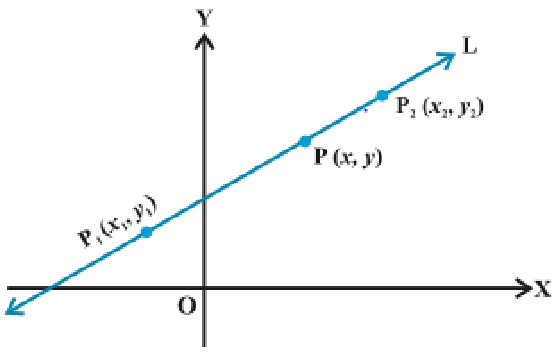
Then by definition of slope of line

$m = \frac{y-y_o}{x-x_o}$.

i.e, $\mathbf{y - y_o = m(x-x_o)}$

This is the equation of the line in Slope-point form.

3.Two- point form



Let $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ be two points passing through the line L. Let P(x,y) be any general point on L.
Since the three points P1, P2 and P are collinear,
slope of PP_1=Slope of P_1P_2

i.e, $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ (OR)

$$\mathbf{y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)}$$

This is the equation of the line in two point form

4.Slope-intercept form

Suppose a line L with slope m cuts the y-axis at a distance ‘c’ from the origin. The distance c is called the y-intercept.
Equation of such straight line is

$$\mathbf{y = mx + c}$$

Simillarly, if the line cuts x-axis at a distance ‘d’ from the origin, then the equation of the line is

y = m(x –d), here d is called the x – intercept.

5.Intercept form

If a line makes x-intercept 'a' and y-intercept 'b' on the axes, then the equation of the line will be of the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

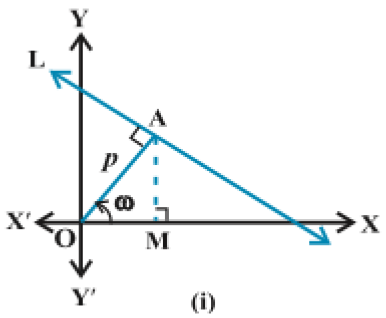
6. Normal Form:

Suppose a non-vertical line is known to us with the following data:

(a) Length of the perpendicular (normal) from the origin to the line.

(b) Angle with the normal makes with the positive direction of x-axis

Let L be the line, whose perpendicular distance from the origin O be OA = p and the angle XOA = ω



Draw a perpendicular AM on the x-axis. We have from the diagram,

OM = p cos ω, and MA = p sin ω

Hence the point

A is (OM,MA) = (p cos ω, p sin ω)

Further L is perpendicular to OA

So, slope of the line L is = $\frac{-1}{slope\ of\ OA} = \frac{-1}{tan\omega} = -\frac{\cos w}{\sin w}$

So, by the slope point form the equation of the line becomes,

y – p sinw = $-\frac{\cos w}{\sin w}$(x – p cos w)

On simplification it becomes

$$\mathbf{x.cos\ w + y\ sin\ w = p}$$

This is the equation of the line in normal form.

Example problems

Example-1:

Find the equation of the line parallel to y-axis and passing through the point (-3,2).

Solution:

Equation of the straight line parallel to x-axis is $x = a$.------(i)

Since this passes through (-3,2)

Equation (i) becomes, $-3 = a$

Hence , equation of the required line is $x = -3$ (OR) $x + 3 = 0$.

Example-2:

Find the equation of the line passing through the point (2,-3) and making an angle 120^o with the positive direction of x- axis.

Solution:

Since the inclination of the line is 120^o,

Slope of the line is $m = \tan 120 = \tan(180-60) = -\tan 60^o = -\sqrt{3}$

Hence , the equation of the line in slope – point form becomes

$$y - (-3) = -\sqrt{3}(x-2) \Rightarrow \sqrt{3}x + y + 3 - 2\sqrt{3} = 0.$$

Example 3:

Find the equation of the line for which $\tan\theta = ½$, where θ is the inclination of the line and (i) y-intercept is -3/2 (ii) x-intercept is 4.

Solution:

(i)Here, slope of the line $m = \tan\theta = ½$, y-intercept $c = -3/2$

Equation of the line is, $y = mx + c$

i.e, $y = (½).x + (-3/2) \Rightarrow 2y = x - 3$ (OR) $x - 2y - 3 = 0$

(ii) If x-intercept $d = 4$,

The equation of the line is $y = m(x-d)$

$\Rightarrow y = (1/2)(x - 4) \Rightarrow 2y = x - 4$ (OR) $x - 2y - 4 = 0$.

Example -4

Find the equation of the line passing through the point (1,2)and making an angle 30^o with y-axis.

Solution:

Let the inclination of the line with x-axis be $'\theta'$

Since the line makes an angle 30^o with y-axis,

$$\theta + 30^o = 90^o \Rightarrow \theta = 60^0$$

So, slope of the line ,m = tan 60^0 = $\sqrt{3}$

Equation of the line is $y-y_1 = m(x-x_1)$

i.e, $y - 2 = \sqrt{3}\,(x-1) \Rightarrow \sqrt{3}\,x - y = 2 - \sqrt{3}$

Example – 5

If A(1,4), B(2,-3) and C(-1,-2)are the vertices of the triangle ABC, find

(i) The equation of median through A

(ii) Equation of the altitude through A

(iii) Equation of the right bisector of the side BC

Solution: (i)

Median is the line joining any vertex of a triangle to the midpoint of the opposite side

Let D be the mid point of the side BC.

Then D = $\left(\frac{2+(-1)}{2}, \frac{-3+(-2)}{2}\right) = \left(\frac{1}{2}, -\frac{5}{2}\right)$

Slope of AD = $\frac{-\frac{5}{2}-4}{\frac{1}{2}-1}$ = 13

∴ The equation of the median through A (i.e, equation of AD) is

$y - 4 = 13(x-1) \Rightarrow 13x - y - 9 = 0$.

(ii) Altitude is the perpendicular line joining any vertex to the opposite side.

Let AM be the altitude from A to the opposite side

Slope of BC = $\frac{-2-(-3)}{-1-2}$ = - $\frac{1}{3}$

Since the altitude AM is perpendicular to BC, Slope of AM = 3

So, equation of AM is y – 4 = 3(x-1) ⇒3x – y + 1= 0.

(iii)Since the right bisector of the side BC is perpendicular to BC,

Slope of BC =3

Also the right bisector BC passes through the midpoint of BC, ie, D$\left(\frac{1}{2}, -\frac{5}{2}\right)$

So the equation is , y –(- $\frac{5}{2}$) = 3(x - $\frac{1}{2}$)

⇒ y + $\frac{5}{2}$ = 3x - $\frac{3}{2}$ ∴3x – y = 4.

Example-6

Find the equation of the line which passes through the point (-4,3) and the portion of the line intercepted between the axes is divided internally in the ratio 5 : 3 by this point

Solution:

Let 'a' and 'b' be the x and y intercept respectively.

Equation of the line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$ ----------- (i)

So the point A(a,0) and B(o,b) be the points where the line meets X and Y axes respectively.

Let P(-4,3) is the point on Ab divided in the ratio 5 : 3

Hence, the point P is $\left(\frac{5*0+3*a}{5+3}, \frac{5*b+3*0}{5+3}\right) = \left(\frac{3a}{8}, \frac{5b}{8}\right)$

Given, $\left(\frac{3a}{8}, \frac{5b}{8}\right)$ = (-4, 3)

∴ $\frac{3a}{8}$ = -4 and $\frac{5b}{8}$ = 8 ⇒ a = - $\frac{32}{3}$, b = $\frac{24}{5}$

Substituting these values in equation (i), we get

$$\frac{x}{\frac{-32}{3}} + \frac{y}{\frac{24}{5}} = 1 \Rightarrow \frac{3x}{-32} + \frac{5y}{24} = 1$$

$\Rightarrow$ -9x + 20y = 96 $\Rightarrow$ 9x – 20y + 96 = 0.

Problems for Practice

Exercise 9.2 complete problems from NCERT text book for class XI mathematics.