CLASS XI

SUBJECT : MATHEMATICS

LESSON: STRAIGHT LINES

MODULE – 2/3

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In this module we will study about

- Various form of equation of straight lines
  - (i) Horizontal and vertical lines
  - (ii) Point - slope form
  - (iii) Slope - intercept form
  - (iv) Two point form
  - (v)Intercept form
  - (vi)Normal form
- Some example problems
- Problems for practice

Various forms of Straight Lines

1.Horizontal and Vertical Lines :

If a horizontal line L is at a distance a from the x-axis, then the ordinate (y-coordinate) of every point lying on the line is either a or –a.

So the equation of the line parallel to x-axis is

$y = a$ or $y = -a$.

The choice of sign will depend upon the position of the line is above or below the y-axis. [Refer figure (a)]

Simillarly,

Equation of the vertical line at a distance b from y-axis is either $x = b$ or $x = -b$[Refer figure (b)]

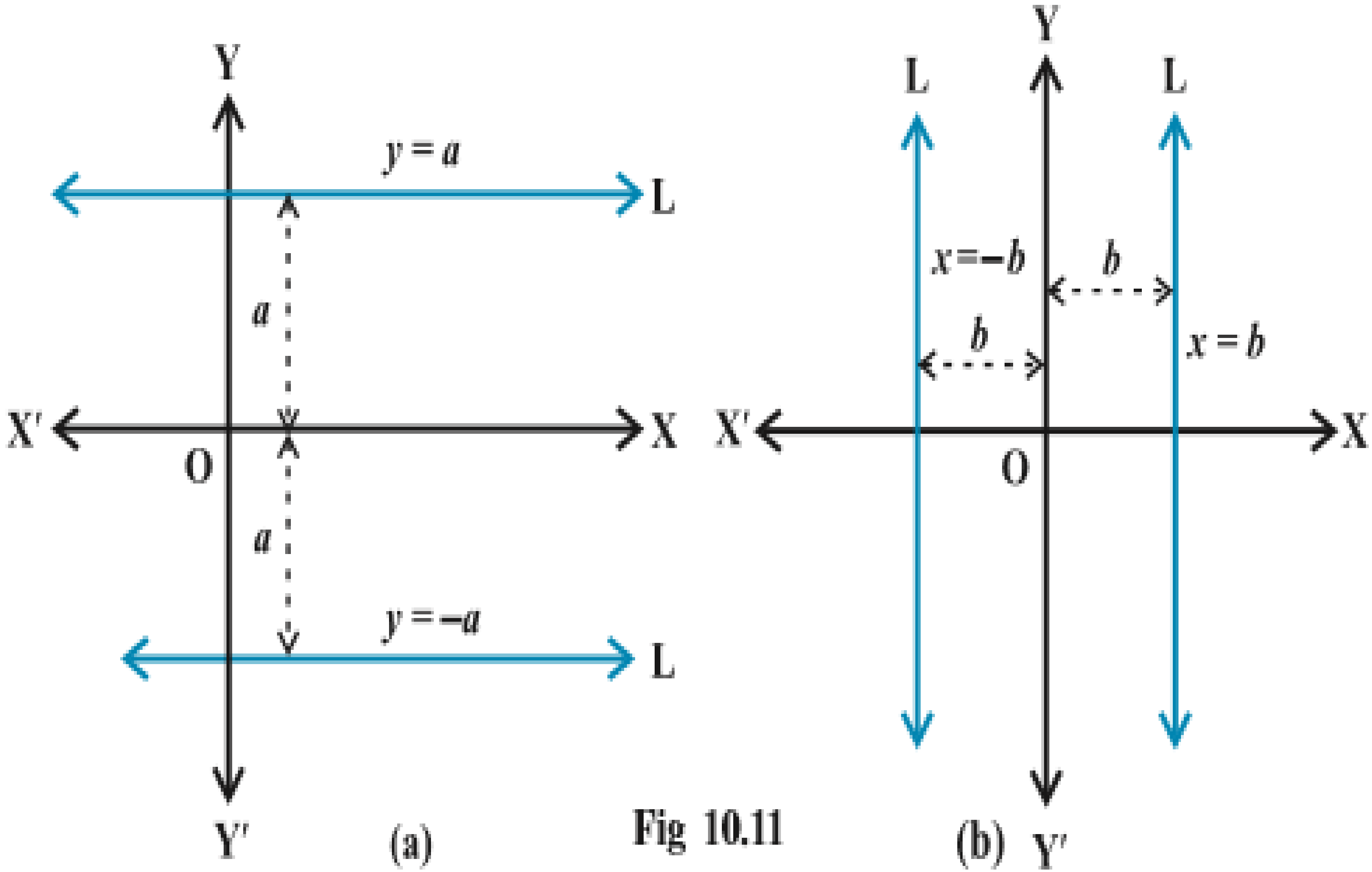
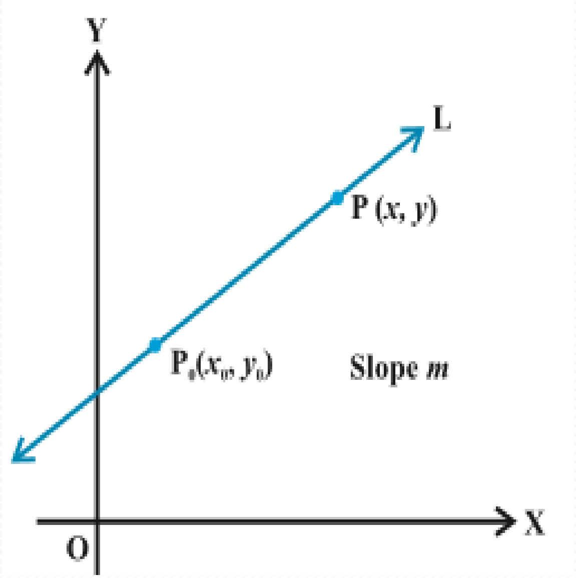


Fig 10.11

2.Point-slope form

Suppose $P_o(x_o,y_o)$is a fixed point on the line L. Let P(x,y) any arbitrary point on the line.



Then by definition of slope of line

$$m = \frac{y-y_o}{x-x_o}.$$

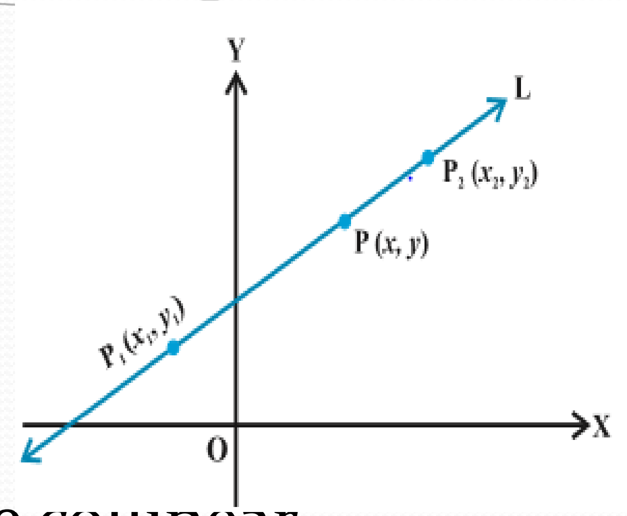
i.e, $\mathbf{y - y_o = m(x-x_o)}$

This is the equation of the line in Slope-point form.

3. Two- point form

Let $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ be two points passing through the line L.

Let P(x,y) be any general point on L.



Since the three points P1,P2 and P are collinear,

Slope of PP_1=Slope of P_1P_2

i.e, $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ (OR)

$$\mathbf{y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)}$$

This is the equation of the line in two point form

4.Slope-intercept form

- Suppose a line L with slope m cuts the y-axis at a distance ‘c’ from the origin. The distance c is called the y-intercept.
- Equation of such straight line is

$$\mathbf{y = mx + c}$$

Simillarly, if the line cuts x-axis at a distance ‘d’ from the origin, then the equation of the line is

$\mathbf{y = m(x - d)}$, here d is called the x – intercept

5.Intercept form

If a line makes x-intercept ‘a’ and y-intercept ‘b’ on the axes, then the

equation of the line will be of the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

6. Normal Form:

Suppose a non vertical line is known to us with the following data:

Length of the perpendicular(normal) from the origin to the line.

Angle with the normal makes with the positive direction of x-axis

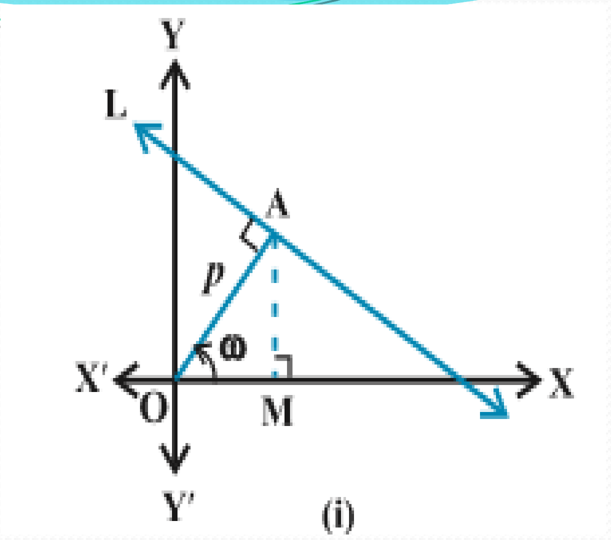
Let L be the line , whose perpendicular distance from the origin O be OA = p and the angle XOA = ω

Draw a perpendicular AM on the x-axis. We have from the diagram,

$OM = p\cos\omega$, and $MA = p\sin\omega$

Hence the point A is $(OM, MA) = (p\cos\omega, p\sin\omega)$

Further L is perpendicular to OA



So, slope of the line L is $= \frac{-1}{slope\ of\ OA} = \frac{-1}{\tan\omega} = -\frac{\cos w}{\sin w}$

So, by the slope point form the equation of the line becomes,

$$y - p\sin w = -\frac{\cos w}{\sin w}(x - p\cos w)$$

On simplification it becomes

$$\mathbf{x.\cos w + y\sin w = p}$$

This is the equation of the line in normal form.

Example problems

Example-1:

Find the equation of the line parallel to y-axis and passing through the point (-3,2).

Solution:

Equation of the straight line parallel to x-axis is

$$x = a. \text{------} (i)$$

Since this passes through (-3,2)

Equation (i) becomes, $-3 = a$

Hence , equation of the required line is

$x = -3$ (OR) $x + 3 = 0$.

Example-2:

Find the equation of the line passing through the point (2,-3) and making an angle 120^{o} with the positive direction of x- axis.

Solution:

Since the inclination of the line is 120^{o},

Slope of the line is $m = \tan 120 = \tan (180-60)$

$$= -\tan 60^{o} = -\sqrt{3}$$

Hence , the equation of the line in slope – point form becomes

$y-(-3) = -\sqrt{3}(x-2) \Rightarrow \sqrt{3}x + y + 3 - 2\sqrt{3} = 0.$

Example 3:

Find the equation of the line for which $\tan\theta = \frac{1}{2}$, where θ is the inclination of the line and (i) y-intercept is $-3/2$ (ii) x-intercept is 4.

Solution:

(i)Here, slope of the line $m = \tan\theta = \frac{1}{2}$

y-intercept $c = -3/2$

Equation of the line is, $y = mx + c$

i.e, $y = (\frac{1}{2}).x + (-3/2) \Rightarrow 2y = x - 3$ (OR) $x - 2y - 3 = 0$

(ii) If x-intercept $d = 4$,

The equation of the line is $y = m(x-d)$

$\Rightarrow y = (1/2)(x - 4) \Rightarrow 2y = x - 4$ (OR) $x - 2y - 4 = 0$.

Example -4

Find the equation of the line passing through the point (1,2)and making an angle 30^o with y-axis.

Solution: Let the inclination of the line with x-axis be $'\theta'$

Since the line makes an angle 30^o with y-axis,

$\theta + 30^o = 90^o \Rightarrow \theta = 60^o$

So, slope of the line ,m = tan $60^o = \sqrt{3}$

Equation of the line is $y-y_1 = m(x-x_1)$

i.e, $y - 2 = \sqrt{3}\,(x-1) \Rightarrow \sqrt{3}\,x - y = 2 - \sqrt{3}$

Example – 5

If A(1,4), B(2,-3) and C(-1,-2)are the vertices of the triangle ABC, find

The equation of median through A

Equation of the altitude through A

Equation of the right bisector of the side BC

Solution: (i)

Median is the line joining any vertex of a triangle to the midpoint of the opposite side

Let D be the mid point of the side BC.

Then D = $\left(\frac{2+(-1)}{2}, \frac{-3+(-2)}{2}\right) = \left(\frac{1}{2}, -\frac{5}{2}\right)$

Slope of AD = $\frac{-\frac{5}{2}-4}{\frac{1}{2}-1}$ = 13

∴ The equation of the median through A (i.e, equation of AD) is

y - 4 = 13(x-1) ⇒13x – y -9 = 0 .

(ii) Altitude is the perpendicular line joining any vertex to the opposite side.

Let AM be the altitude from A to the opposite side

Slope of BC = $\frac{-2-(-3)}{-1-2}$ = $-\frac{1}{3}$

Since the altitude AM is perpendicular to BC, Slope of AM = 3

So, equation of AM is $y - 4 = 3(x-1) \Rightarrow 3x - y + 1 = 0$.

(iii) Since the right bisector of the side BC is perpendicular to BC,

Slope of right bisector of BC =3

Also the right bisector BC passes through the midpoint of BC,

ie, $D\left(\frac{1}{2}, -\frac{5}{2}\right)$

So the equation is of right bisector,

$$y - \left(-\frac{5}{2}\right) = 3\left(x - \frac{1}{2}\right)$$

$$\Rightarrow y + \frac{5}{2} = 3x - \frac{3}{2} \quad \therefore 3x - y = 4.$$

Example-6

Find the equation of the line which passes through the point (-4,3) and the portion of the line intercepted between the axes is divided internally in the ratio 5 : 3 by this point

Solution:

Let ‘a’ and ‘b’ be the x and y intercept respectively.

Equation of the line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$ ----------- (i)

So the point A(a,0) and B(0,b) be the points where the line meets X and Y axes respectively.

Let P(-4,3) is the point on Ab divided in the ratio 5 : 3

Hence, the point P is $\left(\frac{5*0+3*a}{5+3}, \frac{5*b+3*0}{5+3}\right) = \left(\frac{3a}{8}, \frac{5b}{8}\right)$

Given, $\left(\frac{3a}{8}, \frac{5b}{8}\right)$ = (-4, 3)

$\therefore \frac{3a}{8}$ = -4 and $\frac{5b}{8}$ = 3 ⇒ a = - $\frac{32}{3}$, b = $\frac{24}{5}$

Substituting these values in equation (i), we get

$$\frac{x}{\frac{-32}{3}} + \frac{y}{\frac{24}{5}} = 1 \Rightarrow \frac{3x}{-32} + \frac{5y}{24} = 1$$

⇒ -9x + 20y = 96 ⇒ 9x − 20y + 96 = 0.

<u>Problems for Practice</u>

Exercise 9.2 complete problems from NCERT text book for class XI mathematics.