STRAIGHT LINES

MODULE 3/3

In this module we will study about

- General form of equation of straight lines
- Conversion of general equation into different forms
- Distance of a point from a line
- Distance between two parallel lines
- Example problems
- Problems for practice

General Form of a line:

We know that general equation of first degree in two variables is Ax + By + C = 0, where A, B and C are constants. Also graph of the above equation is always a straight line .

Hence we get the general equation of a straight line will be of the form Ax + By + C = 0.

Different forms of Ax + By + C = 0

The general equation of the straight line can be reduced into various form of the equation of the line.

a. Slope-interept form:

If $B \neq 0$, then Ax + By + C = 0 can be written as

$y = -\frac{A}{B}x - \frac{C}{B}$, this is of the form y = mx + c

where $m = -\frac{A}{B} = -\frac{Coefficient\ of\ x}{Coefficient\ of\ y}$ and $C = -\frac{C}{B} = -\frac{Cconstant\ term}{coefficient\ of\ y}$

b. Intercept form:

If $C \neq 0$, then Ax + By + C = 0 can be written as

Ax + By = -C

$\Rightarrow \frac{x}{-\frac{C}{A}} + \frac{y}{-\frac{C}{B}} = 1$, this is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = -\frac{C}{A}$ and $b = -\frac{C}{B}$

c. Normal form:

Let x cosw + y cosw = p-----------(i)
be the normal form of the line represented by the line

Ax + By + C = 0 (or) Ax + By = - C .----------(ii)

If both the equation are same then,

$\frac{A}{cosw} = \frac{B}{sinw} = -\frac{C}{p}$

This gives cosw = $-\frac{Ap}{C}$ and sinw = $-\frac{Bp}{C}$

Now, $\sin^2 w + \cos^2 w = (-\frac{Ap}{C})^2 + (-\frac{Bp}{C})^2$

$\Rightarrow 1 = \frac{p^2(A^2+B^2)}{C^2} \Rightarrow p^2 = \frac{C^2}{(A^2+B^2)}$

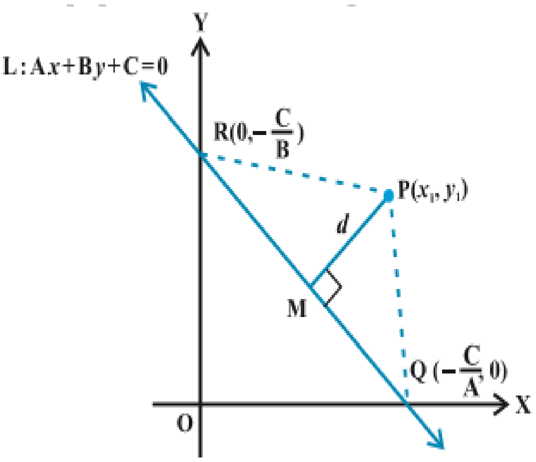
$\Rightarrow p = \pm \frac{C}{\sqrt{A^2+B^2}}$

Therefore, $\cos w = \pm \frac{A}{\sqrt{A^2+B^2}}$ and $\sin w = \pm \frac{B}{\sqrt{A^2+B^2}}$

Thus, the normal form of the equation Ax + By + C = 0 becomes

x cosw + y sinw = p, where cosw,sinw and p can be found from the above

Distance of a point from a line:



Let L be the line Ax + By + C =0 , whose distance from the point $P(x_1,y_1)$ is 'd'. Draw a perpendicular PM from the point p to the line. Let the line meets x and y axes at Q and R respectively.

The coordinates of Q and R are $Q\left(-\frac{C}{A},0\right)$ and $R\left(0,-\frac{C}{B}\right)$

Now, area of triangle PQR is given by

Area(ΔPQR) = $\frac{1}{2}$.PM.QR = $\frac{1}{2}$.d.QR

$\Rightarrow$ d = $\frac{2.\text{Area}(\Delta PQR)}{QR}$ ---------(1)

QR = $\sqrt{\left(0+\frac{C}{A}\right)^2 + \left(\frac{C}{B} - 0\right)^2}$ $= \frac{C}{AB}\sqrt{(A)^2 + (B)^2}$ -------------(2)

Also, Area(ΔPQR) $= \frac{1}{2}\left|x_1\left(0+\frac{C}{B}\right) + \left(-\frac{C}{A}\right)\left(-\frac{C}{B} - y_1\right) + 0(y_1 - 0)\right|$

$= \frac{1}{2}\left|x_1\left(\frac{C}{B}\right) + y_1\left(\frac{C}{A}\right) + \frac{C^2}{AB}\right| = \frac{1}{2}.\frac{C}{AB}|Ax_1 + By_1 + C|$

$\Rightarrow$ 2 Area(ΔPQR) $= \frac{C}{AB}|Ax_1 + By_1 + C|$-----------(3)

Substituting the value of (2) and (3) in (1) we get ,

d = $\frac{2.\text{Area}(\Delta PQR)}{QR}$ $\Rightarrow$ d = $\frac{\frac{C}{AB}|Ax_1 + By_1 + C|}{\frac{C}{AB}\sqrt{(A)^2+(B)^2}}$ $= \frac{|Ax_1 + By_1 + C|}{\sqrt{(A)^2+(B)^2}}$

Thus , the distance from a point P(x_1,y_1) to the line Ax + By + C =0 is

d = $\frac{|\boldsymbol{Ax_1 + By_1 + C}|}{\sqrt{(\boldsymbol{A})^2+(\boldsymbol{B})^2}}$

<u>Note</u>: If the point P is the origin(0,0) the distance becomes,

d = $\frac{|\boldsymbol{A(0) + B(0) + C}|}{\sqrt{(\boldsymbol{A})^2+(\boldsymbol{B})^2}}$ **=** $\frac{|\boldsymbol{C}|}{\sqrt{(\boldsymbol{A})^2+(\boldsymbol{B})^2}}$

<u>Distance between two parallel lines</u>

Let Ax + By + C1 = 0 and Ax + By + C2= 0 be two parallel lines.
(Two parallel lines differ only in their constant terms as their slopes are equal)

The distance between the parallel lines is

d = $\frac{|\boldsymbol{C1 - C2}|}{\sqrt{(\boldsymbol{A})^2+(\boldsymbol{B})^2}}$

Example problems

Example-1. Find the distance between the parallel lines
$3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$

Solution: Here A = 3, B = -4 ,C1 = 7 and C2 = 5.

Distance between the lines is **d** = $\frac{|C1-C2|}{\sqrt{(A)^2+(B)^2}} = \frac{|7-5|}{\sqrt{(3)^2+(-4)^2}} = \frac{2}{5}$.

Example -2

Find the equation of the line perpendicular to the line $x - 2y + 3 = 0$ and passing through the point (1,-2).

Solution:

Given line is $x - 2y + 3 = 0$

Slope of the line $M = -\frac{coeff.of\ x}{coeff.of\ y} = -\frac{1}{-2} = \frac{1}{2}$

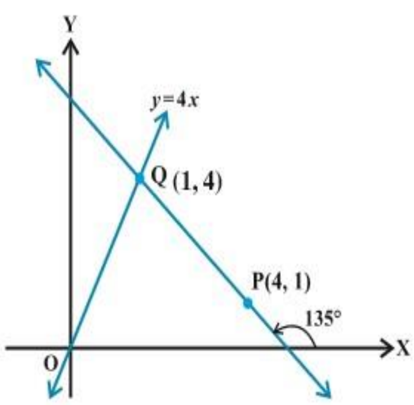
Since the line is perpendicular to the given line,
slope of the required line is m = -2
So equation of the line is in slope point form

$y-y_1 = m(x-x_1) \Rightarrow y-(-2) = -2(x-1)$
$\Rightarrow y + 2 = -2x + 2$
$\Rightarrow 2x + y = 0$

Which is the required line

Example -3:

Find the distance of the line $4x - y = 0$ from the point P(4,1) measured along the line which is making an angle of 135^o with the positive x-axis



Solution: Given line is $4x-y = 0$ ----------(1)
Equation of the line which makes an angle 135^o with x –axis and passing through the point (4,1) is
$y - 1 = \tan 135^o (x-4)$.
$\Rightarrow y - 1 = -1(x - 4)$
$\Rightarrow x + y - 5 = 0$ ---------------------(2)
Solving (1) and (2) we get the point Q as (1,4)
Given point P is (4,1)
So from the problem, required distance

$PQ = \sqrt{(1-4)^2 + (4-1)^2} = \sqrt{9+9}$

$= 3\sqrt{2}$ units.

<u>Example -4:</u>

Find the value of k so that the line 2x + ky – 9 = 0 may be

(i) Parallel to 3x – 4y + 7 = 0

(ii) Perpendicular to 3y + 2x – 1 = 0

Solution:

Given line is 2x + ky – 9 = 0

Slope of the line is M = $-\frac{2}{k}$ **-----------(1)**

(i). Slope of the line 3x – 4y + 7 = 0 is m = $\frac{3}{4}$ ----------(2)

Since the two lines are parallel M = m

$\Rightarrow -\frac{2}{k} = \frac{3}{4} \quad \Rightarrow k = -\frac{8}{3}$

(ii). Slope of the line 3x – 2y – 1 = 0 is**, m_1= -** $\frac{2}{3}$ ---------(3)

Since the given line is perpendicular to (3)

$M.m_1 = -1$

$\Rightarrow (-\frac{2}{k}).(-\frac{2}{3}) = -1$

$\Rightarrow 4 = -3k \Rightarrow k = -\frac{4}{3}$

<u>Example-5</u>:

If p is the length of perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$ which makes intercepts 'a' and 'b' with the axes, prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Solution:

Given line is $\frac{x}{a} + \frac{y}{b} - 1 = 0$.------------(i)

Since 'p' is the length of perpendicular from the origin (0,0) to the line (i),

$$p = \frac{|-1|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} \Rightarrow p^2 = \frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}$$

$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$, as required.

Example – 6:

A variable line passes through a fixed point P. The algebraic sum of the perpendiculars dawn from the point A(2,0),B(0,2) and C (1,1) on the line is zero. Find the coordinate of the point P.

Solution:

Let the slope of the line is m and the fixed point P is (x_1,y_1).
So, the equation of the line is $y- y_1 = m(x - x_1)$ -------(1)
The perpendicular distance from A(2,0) to line (1) is

$$d_1 = \left|\frac{0-y_1-m(2-x_1)}{\sqrt{1+m^2}}\right|$$

Similarly,

The perpendicular distance from B(0,2) to line (1) is

$$d_2 = \left|\frac{2-y_1-m(0-x_1)}{\sqrt{1+m^2}}\right| \quad \text{and}$$

distance from C(1,1) to the line (1) is

$$d_3 = \left|\frac{1-y_1-m(1-x_1)}{\sqrt{1+m^2}}\right|$$

According to the problem , $d_1 + d_2 + d_3 = 0$

$$\Rightarrow \quad \left|\frac{-y_1-2m+mx_1+2-y_1+mx_1+1-y_1-m+mx_1)}{\sqrt{1+m^2}}\right| = 0$$

$$\Rightarrow -3y_1 - 3m + 3mx_1 + 3 = 0$$

$$\Rightarrow -y_1 - m + mx_1 + 1 = 0$$

Since the point (1,1,) lies on this , the point P is (1,1).

Problem for Practice:

All problems from Exercise10.3 and miscellaneous exercise from class XI NCERT mathematics Text book.