Handout -Class XI, Chapter-8, Gravitation :

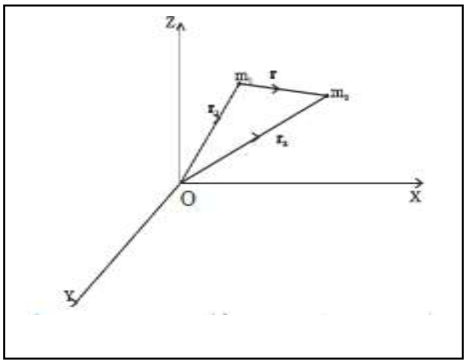
(1/3: Introduction, Universal law of gravitation, Gravitational constant)

- Italian Physicist Galileo (1564-1642) who recognized the fact that all bodies irrespective of their masses are attracted towards the earth with a constant acceleration.
- Galileo Galilei (then professor of mathematics at the University of Pisa) made public demonstrations at the top of Pisa Tower to convince the public.
- Between 1589 and 1592 is said to have dropped two spheres of different masses from the Leaning Tower of Pisa to demonstrate that their time of descent was independent of their mass, according to a biography of Galileo.
- Galileo did experiments with bodies rolling down on inclined planes to find the value of acceleration due to gravity which is close to more accurate value obtained later.
- Using a water clock, Galileo proved that $s \alpha t^2$ and a = gh/l where h and l are the height and length of the inclined plane.
- The earliest recorded model for planetary motion proposed by Ptolemy about 2000 years ago was "Geocentric model".
- "Geocentric model" in which all celestial objects, stars , the sun and the planets all revolved around the earth.
- Indian scientist "Aryabhatta" (5^{th} Century AD) proposed heliocentric model, in which all planets including earth revolve around the sun..
- Aryabhata was the first unmanned Earth satellite built by India, assembled at Peenya, near Bangalore, but launched from the Soviet Union by a Russian-made rocket in 1975.
- A thousand years later, a Polish monk, Nicolas Copernicus (1473-1543) proposed the same theory.
- This theory was discredited by the church. Being a notable supporter of this theory, Galileo faced prosecution from the state for his beliefs.
- At around the same time as Galileo, a Denmark scientist Tycho Brahe (1546-1601) recorded lot of observations of planets with naked eye.
- His compilation was later analyzed by his assistant Johannes Kepler (1571-1640) and later known as Kepler's Laws of planetary motion-3 Laws
- These laws were known to Newton and enabled him to make a great leap in proposing his universal Law of Gravitation.
- Newton explained terrestrial gravitation and Kepler's laws with this Universal Law of Gravitation as follows……..
- The moon revolving in an orbit of radius R_m, has centripetal acceleration due to earth's gravity of magnitude

$$a_m = \frac{v^2}{R_m} = \left(\frac{2\pi R_m}{T}\right)^2 \times \frac{1}{R_m} = \frac{4\pi^2 R_m}{T^2}$$

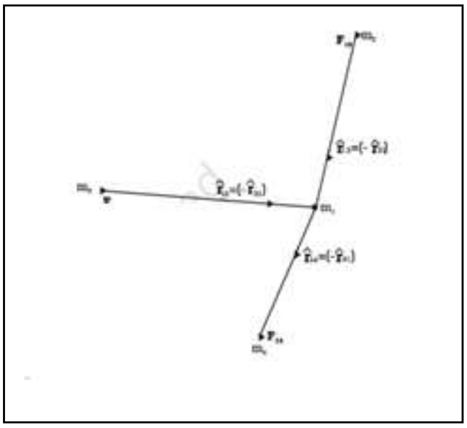
- where v is the speed of the moon related to the time period T by the relation, $v = \frac{2\pi R_m}{T}$
- T = 27.3 days , Rm = $3.84*10^8$m
- a_m =0.0027 m/s^2 <<<< 9.8 m/s^2 (g on the surface of earth)
- This shows that the force due to earth's gravity decreases with distance.
- Acceleration of moon due to earth, $a_m \alpha R_m^{-2}$, and acceleration due to gravity , $g \alpha R_e^{-2}$
- $\frac{g}{a_m} = \frac{R_m^2}{R_e^2} = \frac{(3.84*10^8)^2}{(6.37*106)^2} = 3600, \quad Which\ is\ also\ equal\ to\ \frac{9.8}{0.0027}$

- Newton discovered that the acceleration of the Moon is 1/3,600 smaller than the acceleration at the surface of Earth, he related the number 3,600 to the square of the radius of Earth
- Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- The force F on a point mass m_2 due to another point mass m_1 has the magnitude



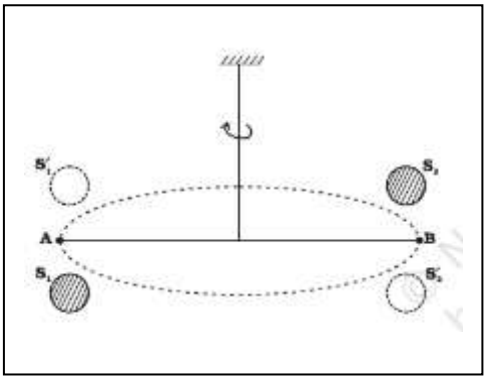
- $|\vec{F}| = G\,\frac{m_1 m_2}{r^2}$
- In vector form,
- $\vec{F} = G\,\frac{m_1 m_2}{r^2}\,(-\widehat{r})$ or
- $\vec{F} = -G\,\frac{m_1 m_2}{r^2}\,(\widehat{r})$ OR $\vec{F} = G\,\frac{m_1 m_2}{r^3}\,(-\vec{r})$

  & G = 6.67 * 10^{-11} Nm^2/Kg^2



- where G is the gravitational constant, $\hat{r}$ is the unit vector from m_1 to m_2 and $\vec{r} = \vec{r_2} - \vec{r_1}$
- *The gravitational force is always attractive. The force F is always along –r.*
- *The force on m_1 due to m_2 is –F by Newton's 3rd Law.*
- F_{12}=-F_{21}
- *F_{12} :Force on the body 1 due to 2, and F_{21} is the force on the body 2 due to 1*
- If we have a collection of point masses, the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses as shown in Fig
- The Total force , $\vec{F_1} = G\,\frac{m_2 m_1}{r21^2}\,\widehat{r_{21}} + G\,\frac{m_3 m_1}{r31^2}\,\widehat{r_{31}} + G\,\frac{m_4 m_1}{r41^2}\,\widehat{r_{41}}$

- The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.
- The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero
- English scientist Henry Cavendish in 1798 determined the value of G experimentally.
- Schematic diagram: S_1 & S_2 are large spheres which are kept on either side of the masses at A &B. (Shaded). When the big masses are taken to the other side of the masses (dotted) the bar AB rotates a little and this angle of rotation can be measured.



- If F be the force of attraction between the big and small spheres, the suspended wire get twisted due to the torque developed, and the restoring torque will be equal to the gravitational torque.
- $F = G\frac{Mm}{d^2}$, If L be the length of the bar AB,
- then torque, $\tau = F * L = G\frac{Mm}{d^2} * L = \tau\theta$
- The value calculated by Cavendish is G = $6.67*10^{-11} Nm^2/Kg^2$

- **References:**
- NCERT Class XI Physics Vol. I
- https://www.nist.gov/blogs/taking-measure/tangled-tale-nists-newton-apple-tree
- https://en.wikipedia.org/
- https://www.youtube.com/watch?v=EE9TMwXnx-s
- https://www.britannica.com/science/gravity-physics/Newtons-law-of-gravity