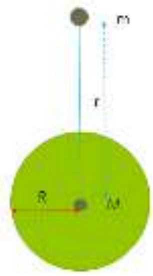
Handout -Class XI, Chapter-8, Gravitation

(2/3: Acceleration due to gravity and its variation with altitude and depth. Gravitational Potential Energy and Gravitational potential)

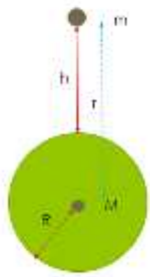
- If we consider earth as a sphere of radius R (6371 km) and mass M (5.97*10^{27} Kg), as a point object so that the entire mass is concentrated at the centre, then the gravitational force of attraction by the earth on that object of mass m will be



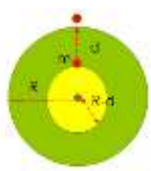
- $F = G\frac{Mm}{r^2}$ = ma (Newton's 2nd Law)
- Also $F = G\frac{Mm}{r^2}$ = mg, where g is the acceleration due to gravity (of earth)
- Therefore, g= $G\frac{M}{r^2}$ which is independent of mass of the body. That means earth attracts all objects equally towards the centre.

- **(i) Variation of g with shape (latitude):**
- g= $G\frac{M}{r^2}$
- Actually earth is not a perfect sphere. It is an oblate spheroid. The planet's rotation causes it to bulge at the equator. Earth's polar radius is 6,356 km and at equatorial radius is 6378 km — a difference of 22 km.
- Therefore g_p =9.807 m/s^2 and g_e = 9.77 m/s^2.
- So weight of a person increases as he moves from equator to pole.

- **(ii) Variation of g with altitude:**



- $F = G\frac{Mm}{r^2}$ = mg which gives , g= $G\frac{M}{r^2}$
- On the surface, g= $G\frac{M}{R^2}$
- At a height h, g'= $G\frac{M}{(R+h)^2}$
- $\frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2} = \left(1+\frac{h}{R}\right)^{-2}$
- $g' = g(1-2\frac{h}{R})$ using Binomial expression and neglecting higher order terms because $\frac{h}{R} \ll 1$
- If $|x| < 1$,
- $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots$
- Here, $x = \frac{h}{R}$, h may be few kilometres, but R is 6400 km.
- Therefore $\frac{h}{R} \ll 1$
- Isaac Newton is generally credited with the generalized binomial theorem, valid for any rational exponent.



- **(iii) Variation of g with depth:**
- If a point mass m at a depth d below the surface of earth.

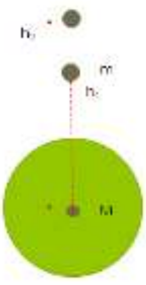
- Its distance from the center of the earth is (R-d).
- Now the earth can be thought of a small sphere of radius (R-d) and a spherical shell of radius, d.
- The force acting on m due to this spherical shell will be cancelled.
- And due to the small sphere alone exists.
- If the entire earth is of uniform density, ρ
- Mass of Earth, M = Volume * Density; $M = \frac{4}{3}\pi R^3 \times \rho$
- g= $G\frac{M}{R^2}$ becomes $g = \frac{G\frac{4}{3}\pi R^3 \times \rho}{R^2} = \frac{4}{3}\pi R\rho G$
- Let the mass m be at a depth d, then,Mass of small earth of radius (R-d) M'= $\frac{4}{3}\pi(R-d)^3 \times \rho$
- g'= $G\frac{M\prime}{(R-d)^2} = \frac{G}{(R-d)^2} \times \frac{4}{3}\pi(R-d)^3 \times \rho \ = g' = \frac{4}{3}\pi G(R-d)\rho$
- $\frac{g\prime}{g} = \frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi R\rho G} = \frac{R-d}{R} = 1 - \frac{d}{R}$
- $g' = g(1 - \frac{d}{R})$
- Hence acceleration due to gravity decreases as depth d increases and at the centre of the earth (d=R), g becomes zero. That means weight of a body becomes zero at the centre of the earth.

- **(iv) Variation of g due to rotation of earth:**

- Earth rotates once in about 24 hours with respect to the Sun about its polar axis from west to east.
- At the poles, the value of g will remain the same whether the Earth is rotating or not. $g' = g$
- But at the equator, g will decrease as per the relation, $g' = g - R\omega^2$ where R is the Radius of the earth ω is the angular speed of earth.
- $\omega = \frac{2\pi}{T} = \frac{2*3.14}{24*60*60} = 7.29 * 10 - 5\, rad/s$

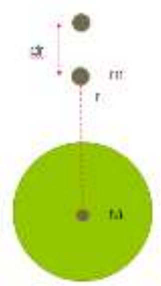
- **Gravitational Potential & Gravitational Potential Energy:**

- Potential energy is the energy stored in the body at a given position or state.
- If the state or positon of the body changes on account of forces acting on it, then the change in PE is just the amount of work done on the body by the force (Work – Energy Theorem)
- Being a conservative force, this work done is independent of the path through which the body is moved.
- The force of gravity is a conservative force and the potential energy of a body arising out of this force is called the gravitational potential energy.



- Force of gravity is practically a constant (mg) at points which are very close to the surface of earth or at distance from the surface much smaller than radius of the earth.
- If we consider two heights h_1 and h_2 above the surface of the earth, the work done in lifting the body vertically from h1 to h2 can be written as

- W_{12} = Force* Displacement
- = mg * (h_2-h_1)
- = mg h_2 – mg h_1
- = W(h_2) – W (h_1)
- Or W(h) = mgh + W_o where W(h) is the PE at a height h and Wo is a constant equal to the PE on the surface of the earth (height, h=0).
- For any arbitrary point,
- $F = \frac{GMm}{r^2}$ where M-mass of the earth, m- mass of the body, r is the distance from the centre of the earth to the particle.
- The work done in lifting the particle from r=r_1 to r= r_2 (r_2>r_1) along a vertical path, we get
- $W_{12} = \int_{r_1}^{r_2} \frac{GMm}{r^2}\, dr \quad = -GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$



- Gravitational Potential Energy W(r) at a distance r,
- W(r) =-$\frac{GMm}{r} + W1$ for r>R
- Also , W_{12} = W(r_2)- W(r_1)
- Setting r = infinity, we get
- W_∞ = W_1 the potential energy at infinity which is set to zero conventionally.
- Therefore Gravitational Potential Energy at a point is just the amount of work done in displacing the particle from infinity to that point.
- W(r) =-$\frac{GMm}{r}$
- Potential energy (gravitational) is zero at infinity. But on the surface of earth it is -$\frac{GMm}{R}$ and at any other point -$\frac{GMm}{r}$ which is always negative.
- That means objects which are taken to infinity and having zero potential energy are moving towards the earth by spending their own energy or lowering their potential energy.
- This case is similar to the electrostatic potential energy of electrons which are revolving in different orbits around the nucleus in the atomic physics chapter. (class XII)
- W(r) =-$\frac{GMm}{r}$
- Here if we handle unit mass (m=1) then we get Gravitational potential at a given height.
- =-$\frac{GM}{r}$
- Gravitational potential energy at a height is equal to gravitational potential at that height * mass of that body.
- = (-$\frac{GM}{r}$) *m
- But $g = \frac{GM}{r^2}$ therefore $\frac{GM}{r} = gr$
- G.P.E = gr*m = gh*m = mgh
- (Here r is the distance from the centre of the earth which is approximated to the height. (class IX)
- References:

- NCERT Class XI Physics Vol. I
- https://en.wikipedia.org/
- https://www.britannica.com/science/gravity-physics/Newtons-law-of-gravity