Class XI Subject- Physics
Chapter–4: Motion in a Plane
Handout of Module 1/2

SCALARS :A scalar quantity is a quantity with magnitude only.

Examples The distance between two points,Mass of an object, The temperature of a body and The time at which a certain event happened.

- **The rules for combining scalars:** are the rules of ordinary algebra.
- Scalars can be added, subtracted, multiplied and divided just as the ordinary numbers

Vector Quantity : It is a quantity that has both a magnitude and a direction

- and obeys the triangle law of addition or equivalently the parallelogram law of addition.
- So, a vector is specified by giving its magnitude by a number and its direction.

Examples Displacement, Velocity, Acceleration and Force etc.

Representation of Vectors

- To represent a vector, we use a bold face type in some books.
- Thus, a velocity vector can be represented by a symbol **v.**
- Since bold face is difficult to produce, when written by hand, a vector is often represented by an arrow placed over a letter, say $\vec{v}$.
- The magnitude of a vector is often called its absolute value, indicated by |**v**| = *v*

Position and Displacement Vectors: Displacement vector is the straight line joining the initial and final positions**Displacement vector is independent on the actual path:**

Equality of Vectors: Two vectors **A** and **B** are said to be equal if, and only if, they have the same magnitude and the same direction.In general, equality is indicated as **A = B**.**MULTIPLICATION OF VECTORS BY REAL NUMBERS:(i) Multiplying a vector A with a positive number:**Multiplying a vector **A** with a positive number n gives a vector whose magnitude is changed by λ the factor □but the direction is the same as that of **A** :

$$|\lambda \mathbf{A}| = \lambda |\mathbf{A}| \text{ if } \lambda > 0.$$

(ii) **Multiplying a vector A by a negative number:**
Multiplying a vector **A** by a negative number λ gives a vector λ**A** whose direction is opposite to the direction of **A** and whose magnitude is λ times |**A**|.

Head-To-Tail Method:vectors are arranged head to tail, this graphical method is called the **head-to-tail method**. The two vectors and their resultant form three sides of a triangle, so this method is also known as **triangle method of vector addition**. . Thus, vector addition is **commutative**: **A** + **B** = **B** + **A** The addition of vectors also obeys the **associative law** .
(**A** + **B**) + **C** = **A** + (**B** + **C**)

Null Vector Or A Zero Vector : Consider two vectors **A** and **A** shown in below figure.

Their sum is **A** + (- **A**). Since the magnitudes of the two vectors are the same, but the directions are opposite, The resultant vector has zero magnitude and is represented by **0** called a **null vector** or a **zero vector** :

- **A** - **A** = **0** |**0**|= 0
- Since the magnitude of a null vector is zero, its direction cannot be specified.
- The null vector also results when we multiply a vector **A** by the number zero.

- The main properties of **0** are :

$$\mathbf{A} + \mathbf{0} = \mathbf{A}$$
$$\lambda\, \mathbf{0} = \mathbf{0}$$
$$\mathbf{0}\, \lambda = \mathbf{0}$$

Physical Meaning Of A Zero Vector:

- When the initial and final positions coincide, the displacement is a .null vector..

Subtraction of vectors:

- We define the difference of two vectors **A** and **B** as the sum of two vectors **A** and **- B**
- **A - B = A + (-B)**

.Law Of Parallelogram For Vector Addition:

It state that if two vectors are represented by two adjacent sides of a parallelogram then magnitude and direction of resultant vector is given by its intersection diagonal

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$ This is known as the **Law of cosines**

$$\tan\alpha = \frac{B\ \sin\theta}{A+B\cos\theta}$$ This equation gives the direction of resultant vector.

Unit vectors: A unit vector is a vector of unit magnitude and points in a particular direction.

$$\hat{r} = \frac{\vec{r}}{|r|}$$ It has no dimension and unit. It is used to specify a direction only.

Rectangular Unit Vectors:

Unit vectors along the *x*-, *y* and *z*-axes of a rectangular coordinate system are denoted by $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ respectively,

- Since these are unit vectors, we have

$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$

- These unit vectors are perpendicular to each other.

RELATIVE VELOCITY IN TWO DIMENSIONS: Suppose that two objects A and B are moving with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ (each with respect to some common frame of reference, say ground.). Then, velocity of object A relative to that of B is :

$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$ and similarly, the velocity of object B *relative to that of* A is :

$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$ Therfore $\mathbf{v}_{AB} = -\mathbf{v}_{BA}$ and $|\mathbf{V}_{AB}| = |V_{BA}|$

The Scalar Product:

A.B = *A* (*B* cos θ) = *B* (*A* cos θ)

Properties of scalar product:

- The scalar product follows the commutative law : **A.B** = **B.A**
- Scalar product obeys the **distributive law**: **A.** (**B** + **C**) = **A.B** + **A.C**
- Further, **A.** (λ **B**) = λ (**A.B**) where λ is a real number.
- For unit vectors I, j,and k we have

For unit vectors **i**,**j**,**k** we have

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$
$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$$

- Given two vectors

A = *Ax* **i** + *Ay* **j** + *Az***k**

B = *Bx* **i** + *By* **j** + *Bz***k**

their scalar product is

A.B=(*Ax* **i** + *Ay* **j** + *Az* **k). (***Bx* **i** + *By* **j** + *Bz* **k)**

= $A_x B_x + A_Y B_Y + A_Z B_Z$

- **A.A**=A^2 since **A.A** = |**A** ||**A**| cos 0 = A^2. **A.B**=0 If **A** and **B** are perpendicular

- **For example:**

- Work is defined as a scalar product of force and displacement . W=**F. d**
- Power is defined as a scalar product of force and velocity. P=**F.v**

Vector Product Of Two Vectors:

- A vector product of two vectors **a** and **b** is a vector **c** such that
- (i) magnitude of **c** = *c* = *ab* sinθ where a and b are magnitudes of **a** and **b** and θ is the angle between the two vectors.
- (ii) **c** is perpendicular to the plane containing **a** and **b**.

Properties Of Vector Product:

- The vector product is **not commutative**, i.e. **a** × **b** ≠ **b** × **a**
- We have **a** × **b** = −**b** × **a**
- Both scalar and vector products are **distributive** with respect to vector addition.
- Thus, **a**.(**b** + **c**) = **a**.**b** + **a**.**c** **a** × (**b** + **c**) = **a** × **b** + **a** × **c**
- From this follow the results ˆ**i** × ˆ**i** = **0**, ˆ**j** × ˆ**j** = **0**, **k**ˆ × **k**ˆ = **0**
- ˆ**i** × ˆ**j** = **k**ˆ Note that the magnitude of ˆ**i** × ˆ**j** is $\sin 90^0$ or 1,
- . Similarly ˆ**j** × **k**ˆ = ˆ**i** and **k**ˆ × ˆ**i** = ˆ**j**
- From the rule for commutation of the cross product, it follows:
- ˆ**j** × ˆ**i** = −**k**ˆ, **k**ˆ × ˆ**j** = −ˆ**i**, ˆ**i** × **k**ˆ = −ˆ**j**
- if ˆ**i**, **j**,**k**ˆ occur cyclically in the above vector product relation, the vector product is positive.
- if ˆ**i**, ˆ**j**,**k**ˆ do not occur in cyclic order, the vector product is negative.

- The expression for **a** × **b** can be put in a determinant form which is easy to remember.

$$\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}$$

For example: ,

- moment of a force is a vector product of lever arm and force . $\boldsymbol{\tau}=\mathbf{r}\times\mathbf{F}$
- angular momentum is a vector product of position vector and momentum. $\mathbf{l}=\mathbf{r}\times\mathbf{p}$