CLASS XI PHYSICS
Chapter–4: Motion in a Plane
Handout of Module 2/2

MOTION IN A PLANE:

Position Vector and Displacement: The position vector r of a particle P located in a plane with reference to the origin of an *x-y* reference frame is given by

$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ where *x* and *y* are components of r along *x*-, and *y*- axes or simply they are the coordinates of the object.

$$\Delta\mathbf{r} = (x'\hat{\mathbf{i}} + y'\hat{\mathbf{j}}) - (x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$$
$$= \hat{\mathbf{i}}\Delta x + \hat{\mathbf{j}}\Delta y$$

where $\Delta x = x' - x$, $\Delta y = y' - y$

Velocity:

Average velocity is given by

$$\bar{\mathbf{v}} = \vec{\mathbf{v}}_{\mathbf{x}}\mathbf{i} + \vec{\mathbf{v}}_{\mathbf{y}}$$

The **velocity (instantaneous velocity)** is given by the limiting value of the average velocity as the time interval approaches zero :

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$
$$\mathbf{v} = \hat{\mathbf{i}}\frac{dx}{dt} + \hat{\mathbf{j}}\frac{dy}{dt} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}.$$

Therefore, the direction of velocity at any point on the path of an object is tangential to the path at that point and is in the direction of motion.

$$v = \sqrt{v_x^2 + v_y^2}$$

and the direction of **v** is given by the angle θ

$$\tan\theta = \frac{v_y}{v_x}, \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Acceleration:

- The **average acceleration** a of an object for a time interval Δt moving in *x-y* plane is the change in velocity divided by the time interval :
  $$\bar{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}.$$
- The acceleration **(instantaneous acceleration)** is the limiting value of the average acceleration as the time interval approaches zero :
  $$\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$$

MOTION IN A PLANE WITH CONSTANT ACCELERATION

- The displacement is the average velocity multiplied by the time interval :

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

Above Equation (can be written in component form as

$$x = x_0 + v_{ox}t + \frac{1}{2}a_x t^2$$

$$y = y_o + v_{oy}t + \frac{1}{2}a_y t^2$$

PROJECTILE MOTION:

- . An object that is in flight after being thrown or projected is called a projectile.
- Such a projectile might be a football, a cricket ball, a baseball or any other object.

Equation of path of a projectile:

- The path of the projectile is always **parabola.**

Time Of Maximum Height: $T_f = 2\,(v_o \sin\theta_o)/g$ of the projectile.

Maximum Height Of A Projectile:

$$\text{Or.}\quad h_m = \frac{(v_o \sin\theta_o)^2}{2g}$$

Horizontal Range Of A Projectile:

- The horizontal distance travelled by a projectile from its initial position ($x = y = 0$) to the position where it passes $y = 0$ during its fall is called the **horizontal range, *R***.
- It is the distance travelled during the time of flight T_f .
- Therefore, the range R is

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

  - R is maximum when $\sin 2\theta_0$ is maximum, i.e., when $\theta_0 = 45^0$.
  - The maximum horizontal range is, therefore,

$$R_m = \frac{v_0^2}{g}$$

UNIFORM CIRCULAR MOTION:

- When an object follows a circular path at a constant speed, the motion of the object is called **uniform circular motion**.
- The word “uniform” refers to the speed, which is uniform (constant) throughout the motion. Therefore, the centripetal acceleration a_c is :

$$a_c = \left(\frac{v}{R}\right)v = v^2/R$$

THANK YOU