Module – 1/4

SQUARES AND SQUARE ROOTS

You can download the chapter of NCERT textbook → http://ncert.nic.in/textbook/textbook.htm?hemh1=6-16

We know that

The area of a rectangle = length × breadth $(l \times b)$

And the area of a square = side × side

Now try to find the area of the following figures

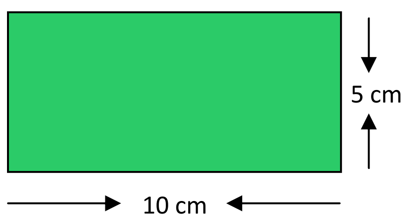

Figure (i)

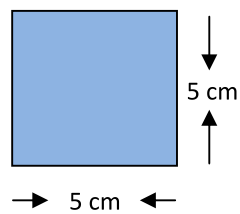

Figure (ii)

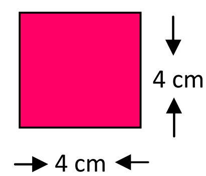

Figure (iii)

In Figure (i) [GREEN]

It is a rectangle.

Here length (l) = 10 cm

and breadth (b) = 5 cm

Area of rectangle = $l \times b$

$= 10\ cm \times 5\ cm$

$= 50\ cm^2$

In Figure (ii) [BLUE]

It is a square.

Here length (l) = 5 cm

and breadth (b) = 5 cm

$l = b = side\ (s)$

Area of square = $side \times side$

$= 5\ cm \times 5\ cm = 25\ cm^2$

In Figure (iii) [RED]

It is also a square.

Here length (l) = 4 cm

and breadth (b) = 4 cm

$l = b = side\ (s)$

Area of square = $side \times side$

$= 4\ cm \times 4\ cm = 16\ cm^2$

Based on figure (ii) [BLUE] and figure (iii) [RED], we can find the area of a square with given side.

The table for the area of a square with given side

Side of square (in cm)	Area of square (in cm^2)	Side of square (in cm)	Area of square (in cm^2)
1	$1 \times 1 = 1 = 1^2$	**6**	$6 \times 6 = 36 = 6^2$
2	$2 \times 2 = 4 = 2^2$	**7**	$7 \times 7 = 49 = 7^2$
3	$3 \times 3 = 9 = 3^2$	**8**	$8 \times 8 = 64 = 8^2$
4	$4 \times 4 = 16 = 4^2$	x	$x \times x = x^2$
5	$5 \times 5 = 25 = 5^2$	a	$a \times a = a^2$

Such numbers like 1, 4, 9, 16, 25, 36, 49, ... are known as **square numbers**.

In general, if a natural number ***m can be expressed as n^2***, where n is also a natural number, then m is a **square number or perfect square**.

Example → $25 = 5^2$, here ***25 can be expressed as 5^2***, so 25 is a square number.

Look here → **1** $= 1^2$, **4** $= 2^2$, **9** $= 3^2$, **16** $= 4^2$, **36** $= 6^2$,
49 $= 7^2$ **64** $= 8^2$ **81** $= 9^2$ **100** $= 10^2$, ……….
In these, **1, 4, 9, 16, 36, 49,** ……… are square numbers.

Properties of Square Numbers

By looking the squares of numbers, easily we can find the properties of square numbers

Number	Square	Number	Square	Number	Square
1	**1**	11	12**1**	21	44**1**
2	**4**	12	14**4**	22	48**4**
3	**9**	13	16**9**	23	52**9**
4	1**6**	14	19**6**	24	57**6**
5	2**5**	15	22**5**	25	62**5**
6	3**6**	16	25**6**	26	67**6**
7	4**9**	17	28**9**	27	72**9**
8	6**4**	18	32**4**	28	78**4**
9	8**1**	19	36**1**	29	84**1**
10	10**0**	20	40**0**	30	90**0**

All the square numbers end with **0, 1, 4, 5, 6 or 9** at units place. None of these end with 2, 3, 7 or 8 at unit's place.

Property – 1 → All the square number end with 0, 1, 4, 5, 6 or 9 at unit place.

Study the same table again by separating some numbers

Number	Square	Number	Square	Number	Square	Unit place in squares
1	**1**	11	12**1**	21	44**1**	**1**
9	8**1**	19	36**1**	29	84**1**	
2	**4**	12	14**4**	22	48**4**	**4**
8	6**4**	18	32**4**	28	78**4**	
3	**9**	13	16**9**	23	52**9**	**9**
7	4**9**	17	28**9**	27	72**9**	
4	1**6**	14	19**6**	24	57**6**	**6**
6	3**6**	16	25**6**	26	67**6**	
5	2**5**	15	22**5**	25	62**5**	**5**
10	10**0**	20	40**0**	30	90**0**	**0**

Property – 2 → The one's place of square depends on the one's place of the numbers.

The one's place of square is **1** for the numbers ends with **1 & 9**.

The one's place of square is **4** for the numbers ends with **2 & 8**.

The one's place of square is **9** for the numbers ends with **3 & 7**.

The one's place of square is **6** for the numbers ends with **4 & 6**.

The one's place of square is **5** for the numbers ends with **5**.

The one's place of square is **0** for the numbers ends with **0**.

Property – 3 → If a number contains some zeros at the end, its square have double zeros.

Table for the square of a number having zero or zeros at the end

Number	Square
10 (**1 zero**)	100 (**2 zeros**)
200 (**2 zeros**)	40,000 (**4 zeros**)
5,000 (**3 zeros**)	2,50,00,000 (**6 zeros**)
70,000 (**4 zeros**)	4900000000 (**8 zeros**)
80,00,000 (**6 zeros**)	6,40,00,00,00,00,000 (**12 zeros**)

Property – 4 → Total natural numbers between two consecutive squares is double of the smaller number

Natural numbers are → **1**, 2, 3, **4**, 5, 6, 7, 8, **9**, 10, 11, 12, 13, 14, 15, **16**, 17, 18, 19, 20, 21, 22, 23, 24, **25**, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, **36**, 37, 38, ………

Between $\underline{\mathbf{1}}^2$ and 2^2 there are two ($\underline{\mathbf{1}} \times 2 = 2$) non square numbers 2, 3.

Between $\underline{\mathbf{2}}^2$ and 3^2 there are four ($\underline{\mathbf{2}} \times 2 = 4$) non square numbers 5, 6, 7, 8.

Between $\underline{\mathbf{3}}^2$ and 4^2 there are six ($\underline{\mathbf{3}} \times 2 = 6$) non square numbers 10, 11, 12, 13, 14, 15.

Between $\underline{\mathbf{4}}^2$ and 5^2 there are eight ($\underline{\mathbf{4}} \times 2 = 8$) non square numbers.

Between $\underline{\mathbf{5}}^2$ and 6^2 there are ten ($\underline{\mathbf{5}} \times 2 = 10$) non square numbers.

Between $\underline{\mathbf{9}}^2$ and 10^2 there are eighteen ($\underline{\mathbf{9}} \times 2 = 18$) non square numbers.

Between $\underline{\mathbf{15}}^2$ and 16^2 there are thirty ($\underline{\mathbf{15}} \times 2 = 30$) non square numbers.

Between $\underline{\boldsymbol{x}^2}$ and $(x+1)^2$ there are $2x$ ($\underline{\boldsymbol{x}} \times 2 = 2x$) non square numbers.

Property – 5 → Total natural numbers between two consecutive squares is one less than the difference of the squares.

Natural numbers are → **1**, 2, 3, **4**, 5, 6, 7, 8, **9**, 10, 11, 12, 13, 14, 15, **16**, 17, 18, 19, 20, 21, 22, 23, 24, **25**, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, **36**, 37, 38, ………

Between **1** and **4** there are two {(**4** - **1**) – 1} non square numbers.
Between **4** and **9** there are four {(**9** - **4**) – 1} non square numbers.
Between **9** and **16** there are six {(**16** - **9**) – 1} non square numbers.
Between **16** and **25** there are eight {(**25** - **16**) – 1} non square numbers.

Between **81** and **64** there are sixteen {(**81** - **64**) – 1} non square numbers.

Property – 6 → If the result is zero on successive subtraction of odd natural numbers starting from 1 (1, 3, 5, 7, …..) from a number, then the number is a perfect square.

Example – 1.

Consider the number **25**.

Now Successively subtract **1, 3, 5, 7, 9**, ... from it.

25 – **1** = 24,
24 – **3** = 21,
21 – **5** = 16,
16 – **7** = 9,
9 – **9** = **0** **(zero)**

So, 25 is a perfect square.

Example – 2.

Consider the number **38**.

Now Successively subtract **1, 3, 5, 7, 9**, ... from it.

38 – 1 = 37,
37 – 3 = 34,
34 – 5 = 29,
29 – 7 = 22,
22 – 9 = 13,
13 – 11 = 2,
2 – 13 = - 11 ≠ **0**

So, 38 is not a perfect square.

Property – 7 → The sum of first n odd natural numbers is n^2.

Odd numbers are → 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, ……….

Sum of first **1** odd number = 1 = $\underline{\mathbf{1}}^2$

Sum of first **2** odd numbers = 1 + 3 = 4 = $\underline{\mathbf{2}}^2$

Sum of first **3** odd numbers = 1 + 3 + 5 = 9 = $\underline{\mathbf{3}}^2$

Sum of first **4** odd numbers = 1 + 3 + 5 + 7 = 16 = $\underline{\mathbf{4}}^2$

Sum of first **5** odd numbers = 1 + 3 + 5 + 7 + 9 = 25 = $\underline{\mathbf{5}}^2$

Sum of first **9** odd numbers = 1 + 3 + 5 + 7 + 9 + … = ? = $\underline{\mathbf{9}}^2$

Sum of first **18** odd numbers = 1 + 3 + 5 + 7 + 9 + … = ? = $\underline{\mathbf{18}}^2$

Sum of first **n** odd numbers = 1 + 3 + 5 + 7 + 9 + …n terms = $\underline{\mathbf{n}}^2$

Property – 8 → We can express the square of any odd number as the sum of two consecutive positive integers.

$1^2 = 1 = 0 + 1$

$3^2 = 9 = 4 + 5$

$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

$9^2 = 81 = 40 + 41$

$13^2 = 169 = 84 + 85$

$21^2 = 441 = 220 + 221$

Many properties can be understand with the patterns also

- **Product of two consecutive even or odd natural numbers**

$3 \times 5 = (4 - 1) \times (4 + 1) = 4^2 - 1$

$4 \times 6 = (5 - 1) \times (5 + 1) = 5^2 - 1$

$7 \times 9 = (8 - 1) \times (8 + 1) = 8^2 - 1$

$11 \times 13 = (12 - 1) \times (12 + 1) = 12^2 - 1$

$24 \times 26 = (25 - 1) \times (25 + 1) = 25^2 - 1$

- **Some more patterns in square numbers**

$1^2 = 1$

$11^2 = 1\ 2\ 1$

$111^2 = 1\ 2\ 3\ 2\ 1$

$1111^2 = 1\ 2\ 3\ 4\ 3\ 2\ 1$

$11111^2 = 1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$

$11111111^2 = 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1$

- **Another interesting pattern**

$7^2 = 49$

$67^2 = 4489$

$667^2 = 444889$

$6667^2 = 44448889$

$66667^2 = 4444488889$

$666667^2 = 444444888889$

You also try to find some more properties and patterns on square of numbers and discuss with your teachers.

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