Module – 2/4

SQUARES AND SQUARE ROOTS

You can download the chapter of NCERT textbook → http://ncert.nic.in/textbook/textbook.htm?hemh1=6-16

Finding the Square of a Number

Squares of small numbers like 1, 2, 3, 4, 5, 6, 7, ... etc. are easy to find.

$1^2 = 1 \times 1 = 1$

$2^2 = 2 \times 2 = 4$

$3^2 = 3 \times 3 = 9$

$4^2 = 4 \times 4 = 16$

$8^2 = 8 \times 8 = 64$

Now try for the numbers 12, 37 & 45

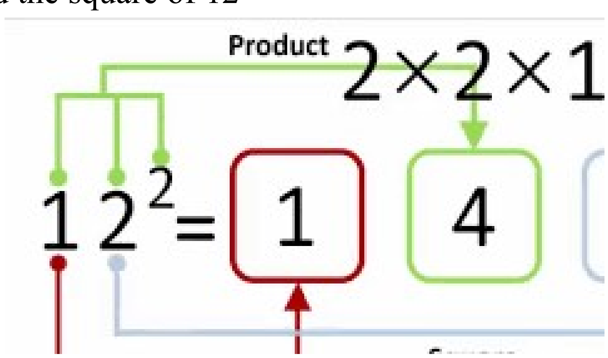
$12^2 = 12 \times 12 = ?$

$37^2 = 27 \times 27 = ?$

$45^2 = 45 \times 45 = ?$ Not easy ?

Now we will find the square of two digit numbers so quickly.

For example, try to find the square of 12



So $12^2 = 144$

Now again try with another example

$37^2 =$

Digits are 3 & 7

3^2	2*3*7	7^2	
↓	↓	↓	
9	42	49	
↓	↓	↓	
9+(4)	2+(4)	9	
↓	↓	↓	
13	6	9	= 1369

So $37^2 = 1369$

- Start thinking from the last digit

Let's try one more example

$56^2 = ?$

Here digits are 5 and 6

Last digit is 6

Step – 1 (square of last digit)

		$6^2 = 36$

Step – 2 (double of product of both the digits)

	2*5*6 = 60 (+3) → (63)	6

Step – 3 (square of first digit)

$5^2 = 25$ (+6) → (31)	3	6

Step – 4 (combining all)

3136

So $56^2 = 3136$

Finding the Square of a Number ending 5

Consider a number with unit digit 5, i.e., a5

$(a5)^2$

$= (10a + 5)^2$

$= (10a + 5)\,(10a + 5)$

$= 10a\,(10a + 5) + 5\,(10a + 5)$

$= 100a^2 + 50a + 50a + 25$

$= 100a\,(a + 1) + 25$

$= a\,(a + 1)$ hundred + 25

So, $(a5)^2$ = a(a + 1) hundred + 25

Using, $(a5)^2$ = a(a + 1) hundred + 25

2$5^2$ = (**2** × 3) hundreds + 25 = 600 + 25 = 625

3$5^2$ = (**3** × 4) hundreds + 25 = 1200 + 25 = 1225

4$5^2$ = (**4** × 5) hundreds + 25 = 2000 + 25 = 2025

7$5^2$ = (**7** × 8) hundreds + 25 = 5600 + 25 = 5625

12$5^2$ = (**12** × 13) hundreds + 25 = 15600 + 25 = 15625

36$5^2$ = (**36** × 37) hundreds + 25 = 133200 + 25 = 133225

Now try some more,

1$5^2$ = (**1** × 2) hundreds + 25 = _____

9$5^2$ = (______) hundreds + 25

135$5^2$ = (______) hundreds + 25

Pythagorean triplets

The set of three numbers are called Pythagorean triplet if the sum of the squares of smaller numbers is equal to the square of largest number

Consider the following

$3^2 + 4^2 = 9 + 16 = 25 = 5^2$

The collection of numbers 3, 4 and 5 is known as **Pythagorean triplet**.

6, 8, 10 is also a Pythagorean triplet, since

$6^2 + 8^2 = 36 + 64 = 100 = 10^2$

Again, observe that

$5^2 + 12^2 = 25 + 144 = 169 = 13^2$.

The numbers 5, 12, 13 form another such triplet.

General form for Pythagorean triplet

For any natural number m > 1

$(m^2 + 1)^2 = (m^2)^2 + 2(m^2) \times 1 + 1^2$

$(m^2 - 1)^2 = (m^2)^2 - 2(m^2) \times 1 + 1^2$

On subtraction

$(m^2 + 1)^2 - (m^2 - 1)^2 = [(m^2)^2 - 2(m^2)*1 + 1^2] - [(m^2)^2 + 2(m^2)*1 + 1^2] = (2m)^2$

$\Rightarrow (m^2 + 1)^2 - (m^2 - 1)^2 = (2m)^2$

$\Rightarrow (m^2 + 1)^2 = (2m)^2 + (m^2 - 1)^2$

here we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$.

So, 2m, $m^2 - 1$ and $m^2 + 1$ forms a Pythagorean triplet.

General form for Pythagorean triplet is 2m, $m^2 - 1$ and $m^2 + 1$

Example : Write a Pythagorean triplet whose smallest member is 8.

Solution: We can get Pythagorean triplets by using general form $2m, m^2 - 1, m^2 + 1$.

Here $m^2 + 1$ cannot be smallest number.

Let us first take $m^2 - 1 = 8$

So, $m^2 = 8 + 1 = 9$ which gives $m = 3$ Therefore, $2m = 6$ and $m^2 + 1 = 10$

The triplet is thus 6, 8, 10. But 8 is not the smallest member of this.

So, let us try $2m = 8$, then $m = 4$ We get $m^2 - 1 = 16 - 1 = 15$ and $m^2 + 1 = 16 + 1 = 17$

The triplet is 8, 15, 17 with 8 as the smallest member.

Example : Find a Pythagorean triplet in which one member is 12.

Solution: If we take $m^2 - 1 = 12$, Then, $m^2 = 12 + 1 = 13$

Then the value of m will not be an integer.

So, we try to take $m^2 + 1 = 12$. Again $m^2 = 11$ will not give an integer value for m.

So, let us take $2m = 12$ then $m = 6$

Thus, $m^2 - 1 = 36 - 1 = 35$ and $m^2 + 1 = 36 + 1 = 37$

Therefore, the required triplet is 12, 35, 37.

- All Pythagorean triplets may not be obtained using this form. For example another triplet 5, 12, 13 also has 12 as a member.
- If you multiply any Pythagorean triplet with any constant, the resulting set will again the Pythagorean triplet.

  Example

  (3, 4, 5) is a Pythagorean triplet

  $(3, 4, 5) \times 2 = (6, 8, 10)$ is also a Pythagorean triplet

  $(3, 4, 5) \times 3 = (9, 12, 15)$ is also a Pythagorean triplet

  $(3, 4, 5) \times 7 = (21, 28, 35)$ is also a Pythagorean triplet

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