Module – 4/4

SQUARES AND SQUARE ROOTS

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Finding square root by division method

When the numbers are large, even the method of finding square root by prime factorization becomes lengthy and difficult. To overcome this problem we use Long Division Method.

For long division method we need to determine the number of digits in the square root.

Observe the following table (Look the Number of digits):

Number	Digits in number	Square	Digits in square
10	2	100	3
15	2	225	3
31	2	961	3
32	2	1024	4
99	2	9801	4
100	3	10000	5
225	3	50625	5
330	3	108900	6
999	3	998001	6

Here if a perfect square is a 3-digit or a 4-digit number, then its square root will have 2-digits & if a perfect square is a 5 or 6-digit number, then its square root will have 3-digits.

Square root by long division method

In long division method, we have to concentrate on various steps:

Consider the following steps to find the square root of 529.

Step 1 Place a bar over every pair of digits starting from the digit at one's place. If the number of digits in it is odd, then the left-most single digit too will have a bar.

Thus we have, $\overline{\mathbf{5}}\ \overline{\mathbf{29}}$.

Step 2 Find the largest number whose square is less than or equal to the number under the extreme left bar ($2^2 < 5 < 3^2$). Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend (here 5). Divide and get the remainder (1 in this case).

$$\begin{array}{r|l} & \quad 2 \\ \hline 2 & \quad \overline{5}\ \overline{29} \\ & -\ 4 \\ \hline & \ \ 1 \end{array}$$

Step 3 Bring down the number under the next bar (i.e., 29 in this case) to the right of the remainder. So the new dividend is 129.

$$\begin{array}{r|l} & \quad 2 \\ \hline 2 & \quad \overline{5}\ \overline{29} \\ & -\ 4 \\ \hline & \ \ 1\ 29 \end{array}$$

Step 4 Double the quotient and enter it with a blank on its right.

$$\begin{array}{r|l} & \quad 2 \\ \hline 2 & \quad \overline{5}\ \overline{29} \\ & -\ 4 \\ \hline 4 & \ \ 1\ 29 \end{array}$$

Step 5 Guess a largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new quotient the product is less than or equal to the dividend.

In this case $42 \times 2 = 84$.

As $43 \times 3 = 129$ so we choose the new digit as 3. Get the remainder.

$$\begin{array}{r|l}
 & \mathbf{23} \\ \hline
\mathbf{2} & \overline{\mathbf{5}}\ \overline{\mathbf{29}} \\
 & \mathbf{-\ 4} \\ \hline
\mathbf{43} & \mathbf{1\ 29} \\
 & \mathbf{-\ 1\ 29} \\ \hline
 & \quad \mathbf{0}
\end{array}$$

Step 6 Since the remainder is 0 and no digits are left in the given number,

therefore, $\sqrt{\mathbf{529}} = \mathbf{23.}$

Now consider another example – to ***find the square root of 116964***

Step 1 Place a bar over every pair of digits starting from the one’s digit.

$\overline{11}\ \overline{69}\ \overline{64}$

Step 2 Find the largest number whose square is less than or equal to the number under the left-most bar ($3^2 < 11 < 4^2$). Take this number as the divisor and the number under the left-most bar as the dividend. Divide and get the remainder i.e., 3 in this case.

$$\begin{array}{r|l}
 & 3 \\ \hline
3 & \overline{11}\ \overline{69}\ \overline{64} \\
 & -\ \ 9 \\ \hline
 & \quad 2
\end{array}$$

Step 3 Bring down the number under the next bar (i.e., 69) to the right of the remainder. The new dividend is 269.

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{11}\ \overline{69}\ \overline{64} \\ & -\ 9 \\ \hline & 2\ 69 \end{array}$$

Step 4 Double the quotient and enter it with a blank on its right.

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{11}\ \overline{69}\ \overline{64} \\ & -\ 9 \\ \hline 6 & 2\ 69 \end{array}$$

Step 5 Guess a largest possible digit to fill the blank which also becomes the new digit in the quotient such that when the new digit is multiplied to the new quotient the product is less than or equal to the dividend. In this case we see that $64 \times 4 = 256$.
So the new digit in the quotient is 4. Get the remainder.

$$\begin{array}{r|l} & 34 \\ \hline 3 & \overline{11}\ \overline{69}\ \overline{64} \\ & -\ 9 \\ \hline 64 & 2\ 69 \\ & -\ 2\ 56 \\ \hline & \quad 13 \end{array}$$

Step 3 Bring down the number under the next bar (i.e., 64) to the right of the remainder. The new dividend is 1364.

$$\begin{array}{r|l} & 34 \\ \hline 3 & \overline{11}\ \overline{69}\ \overline{64} \\ & -\ 9 \\ \hline 64 & 2\ 69 \\ & -\ 2\ 56 \\ \hline & \quad 13\ 64 \end{array}$$

Step 4 Add the new digit (4) in the divisor (i.e., 64 + 4 = 68).

	34
3	$\overline{11}\ \overline{69}\ \overline{64}$
	- 9
64	2 69
	- 2 56
68	13 64

Step 5 Again Guess a largest possible digit to fill the blank which also becomes the new digit in the quotient such that when the new digit is multiplied to the new quotient the product is less than or equal to the dividend. In this case we see that 682 × 2 = 1364.

	342
3	$\overline{11}\ \overline{69}\ \overline{64}$
	- 9
64	2 69
	- 2 56
682	13 64
	- 13 64
	0

Step 6 Since the remainder is 0 and no bar left,

therefore, $\sqrt{\mathbf{116964}} = \mathbf{342}$.

Example:

Find the least number that must be subtracted from 5607 so as to get a perfect square. Also find the square root of the perfect square.

Solution: Let us try to find 5607 by long division method.

$$
\begin{array}{r|l}
 & \quad 74 \\
\hline
7 & \quad \overline{56}\,\overline{07} \\
 & -\ 49 \\
\hline
144 & \quad 7\ 07 \\
 & -\ 5\ 76 \\
\hline
 & \quad 1\ 31
\end{array}
$$

We get the remainder 131. It shows that 74^2 is less than 5607 by 131. This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is 5607 – 131 = 5476.

And, $\sqrt{\mathbf{5476}} = \mathbf{74}$.

Example:

Find the greatest 4-digit number which is a perfect square.

Solution: Greatest number of 4-digits = 9999. We find 9999 by long division method.

$$
\begin{array}{r|l}
 & \quad 99 \\
\hline
9 & \quad \overline{99}\,\overline{99} \\
 & -\ 81 \\
\hline
189 & \quad 18\ 99 \\
 & -\ 17\ 01 \\
\hline
 & \quad\ \ 1\ 98
\end{array}
$$

The remainder is 198. This shows 992 is less than 9999 by 198. This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is 9999 – 198 = 9801.

And, $\sqrt{\mathbf{9801}} = \mathbf{99}$

Square Roots of Decimals

Consider 17.64

Step 1 To find the square root of a decimal number we put bars on the integral part (i.e., 17) of the number in the usual manner. And place bars on the decimal part (i.e., 64) on every pair of digits beginning with the first decimal place. Proceed as usual.
We get $\mathbf{\overline{17}.\overline{64}}$

Step 2 Now proceed in a similar manner. The left most bar is on 17 and $4^2 < 17 < 5^2$. Take this number as the divisor and the number under the left-most bar as the dividend, i.e., 17. Divide and get the remainder.

Step 3 The remainder is 1. Write the number under the next bar (i.e., 64) to the right of this remainder, to get 164.

Step 4 Double the divisor and enter it with a blank on its right. Since 64 is the decimal part so put a decimal point in the quotient.
Step 5 We know 82 × 2 = 164, therefore, the new digit is 2. Divide and get the remainder.

$$\begin{array}{r|l} & \;4.2 \\ \hline 4 & \overline{17}.\overline{64} \\ & -\ 16 \\ \hline 82 & \;\;1\,64 \\ & -\ 1\,64 \\ \hline & \qquad 0 \end{array}$$

Step 6 Since the remainder is 0 and no bar left, therefore $\sqrt{\mathbf{17.64}}$ **= 4.2** .

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