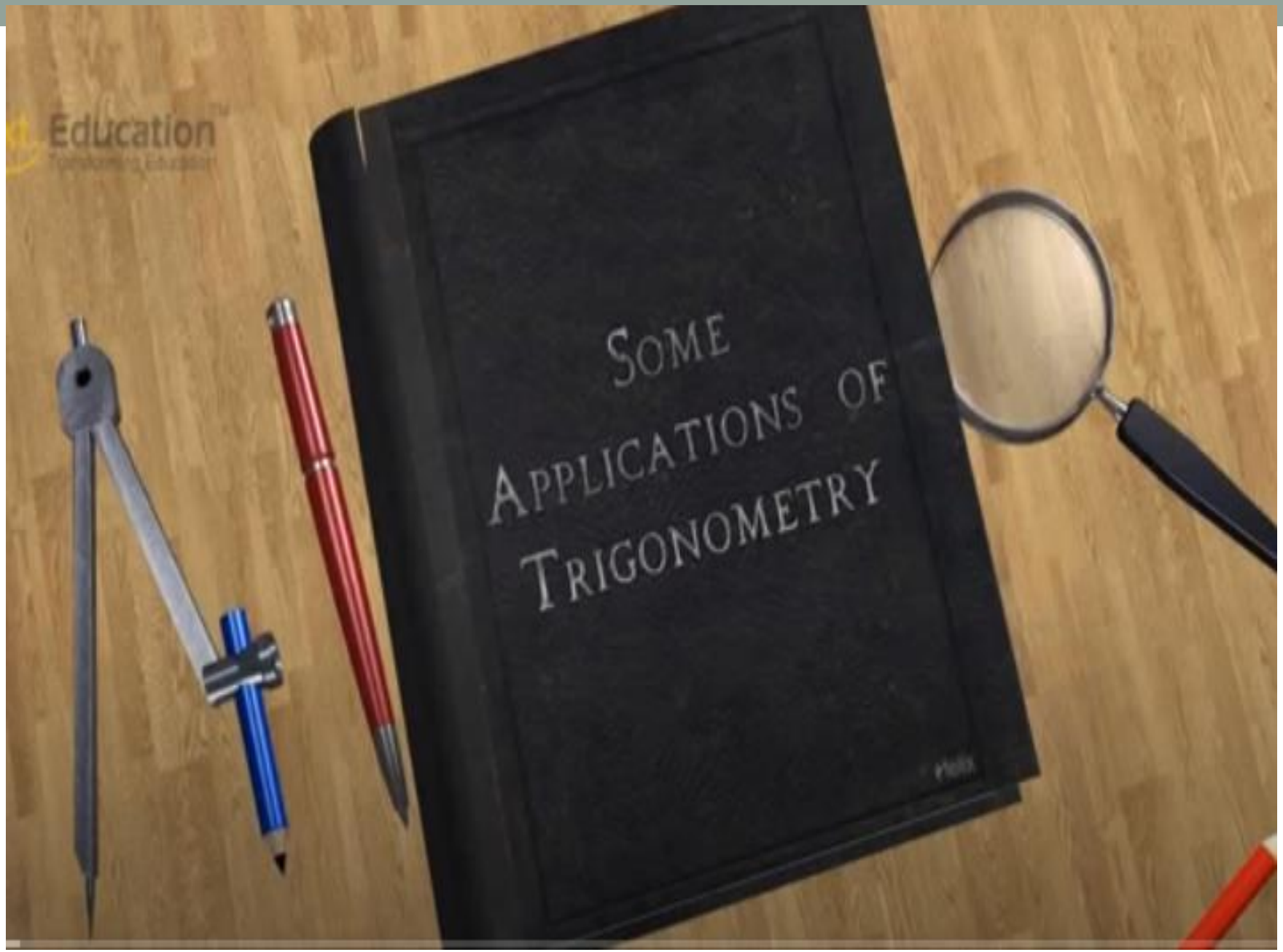
MATHEMATICS
CLASS X


Education
SOME
APPLICATIONS OF
TRIGONOMETRY

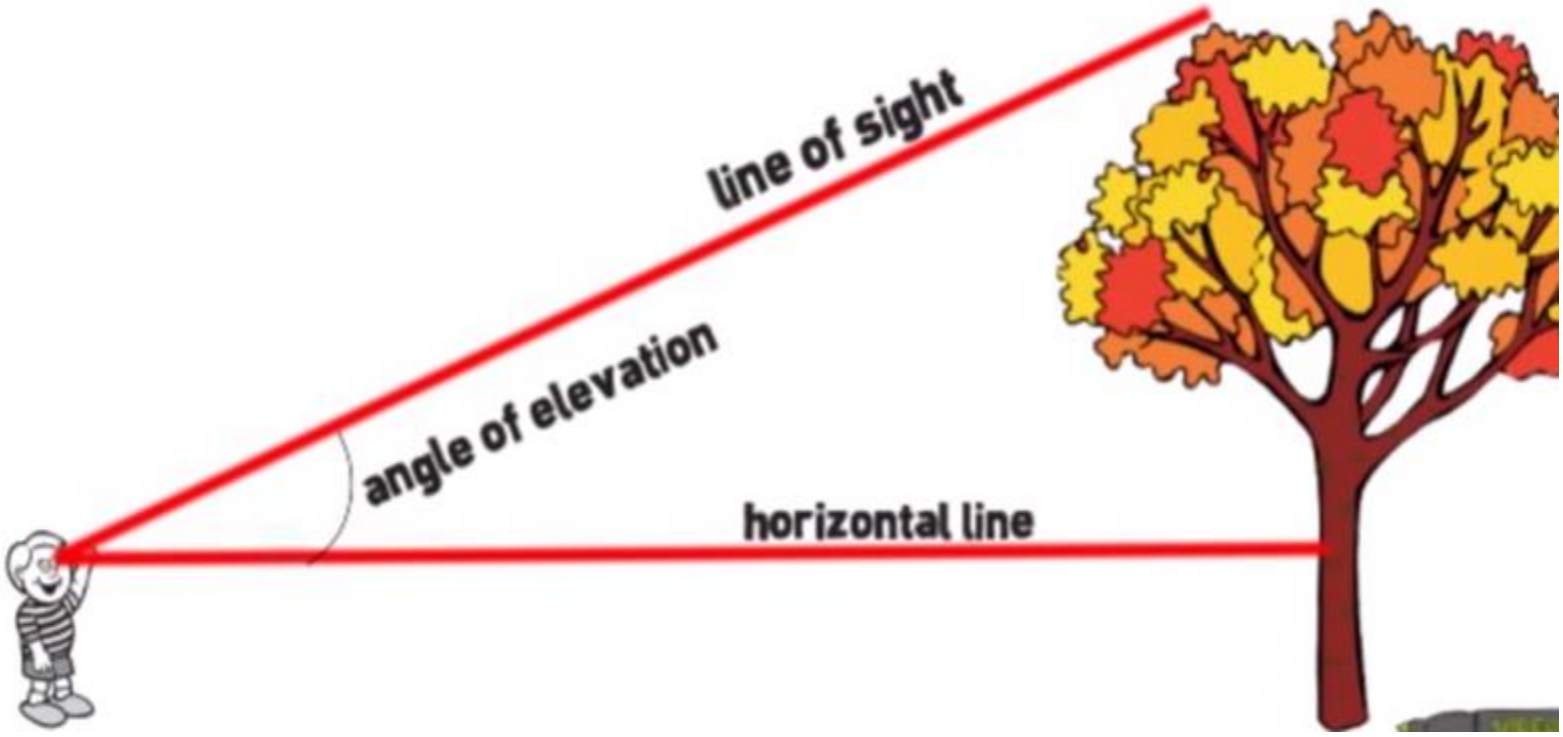

HEIGHTS
AND
DISTANCES

Before knowing what are the Application Of Trigonometry you should know the following terms...

* Line of sight.
* Horizontal level.
* Angle of elevation.
* Angle of depression.

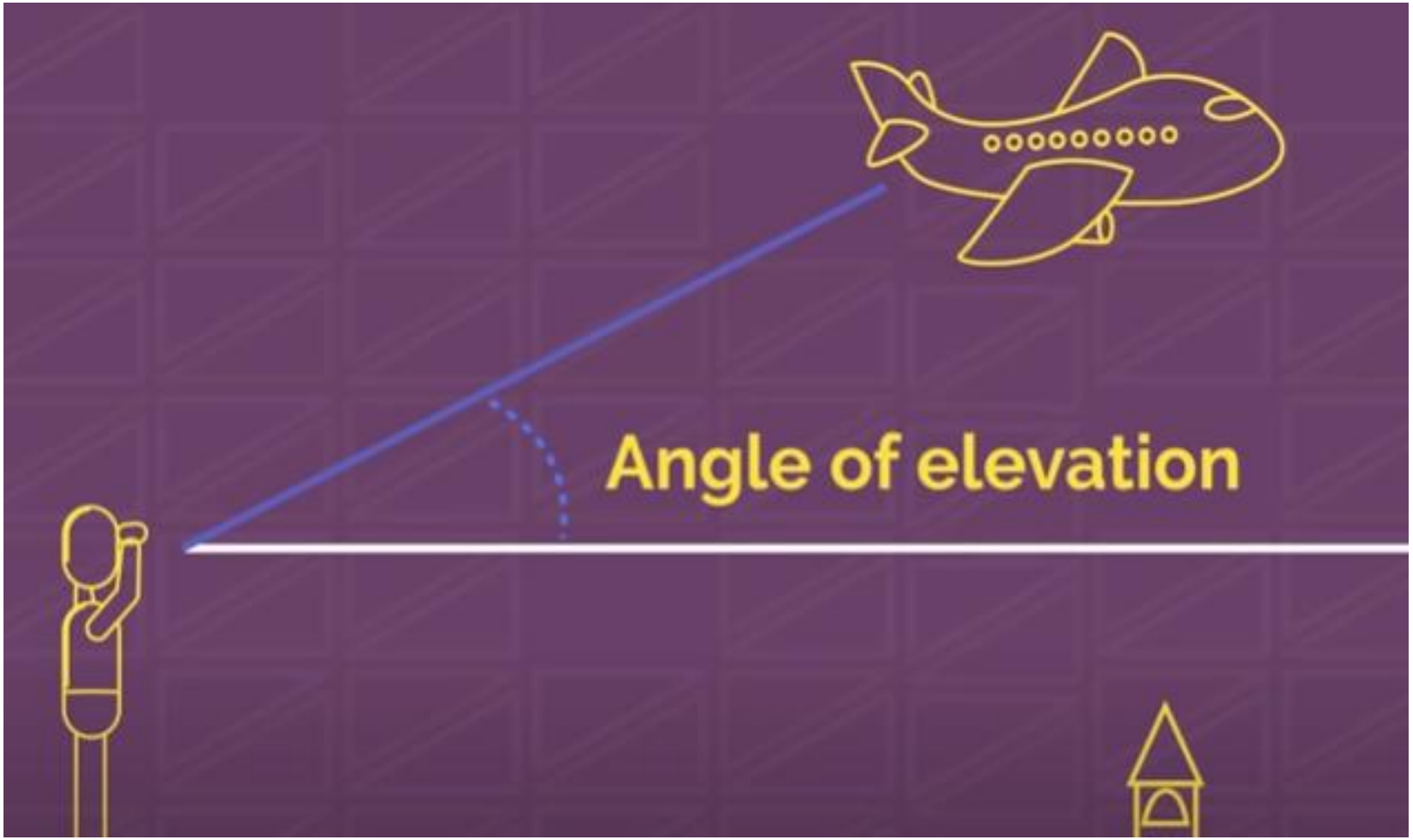
THE LINE DRAWN FROM THE EYE OF THE OBSERVER TO THE POINT TO BE OBSERVED

THE ANGLE MADE BY THE LINE OF SIGHT ABOVE THE HORIZONTAL IS KNOWN AS THE ANGLE OF ELEVATION.

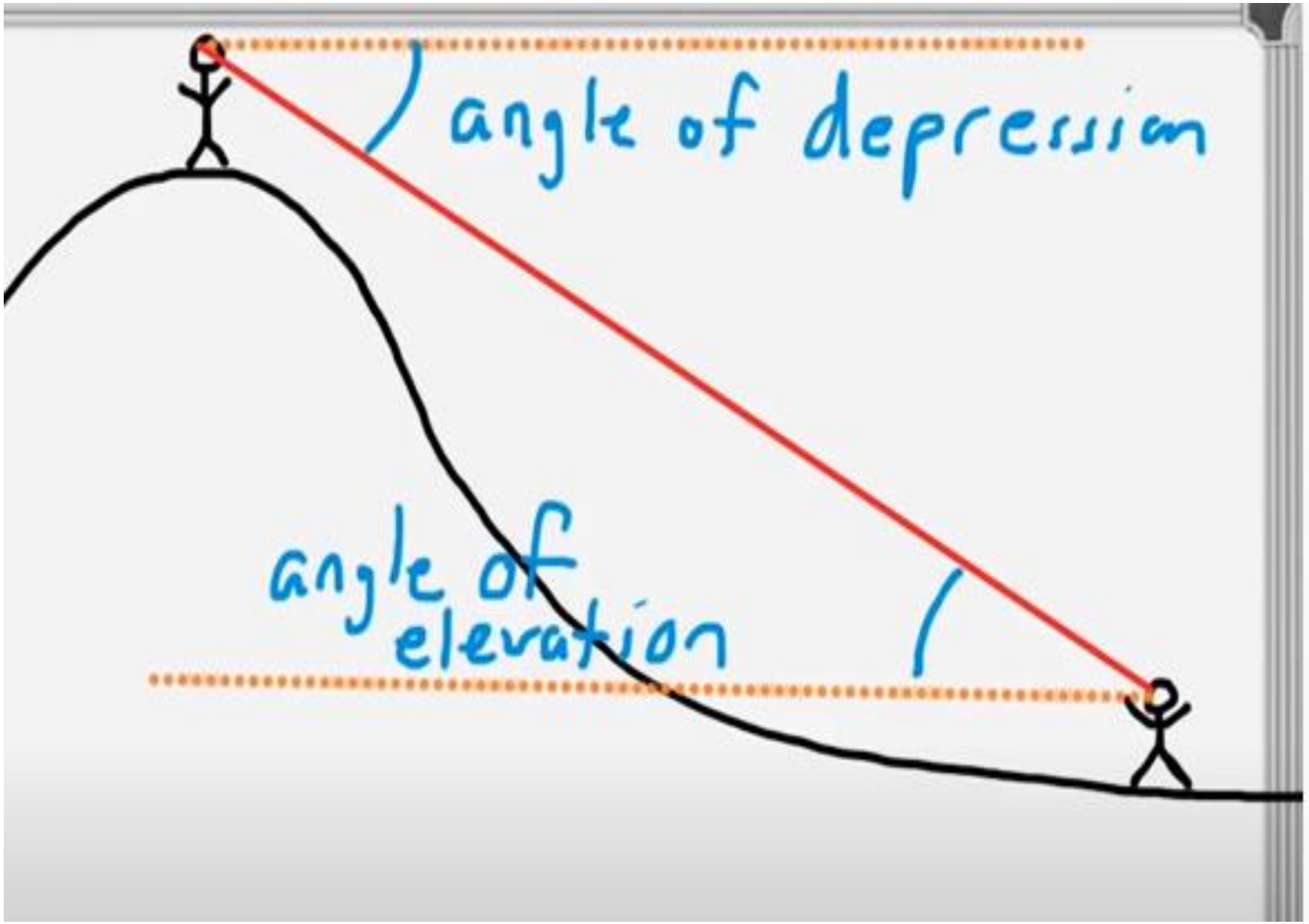


THE ANGLE MADE BY THE LINE OF SIGHT BELOW THE HORIZONTAL LEVEL IS KNOWN AS THE ANGLE OF DEPRESSION.

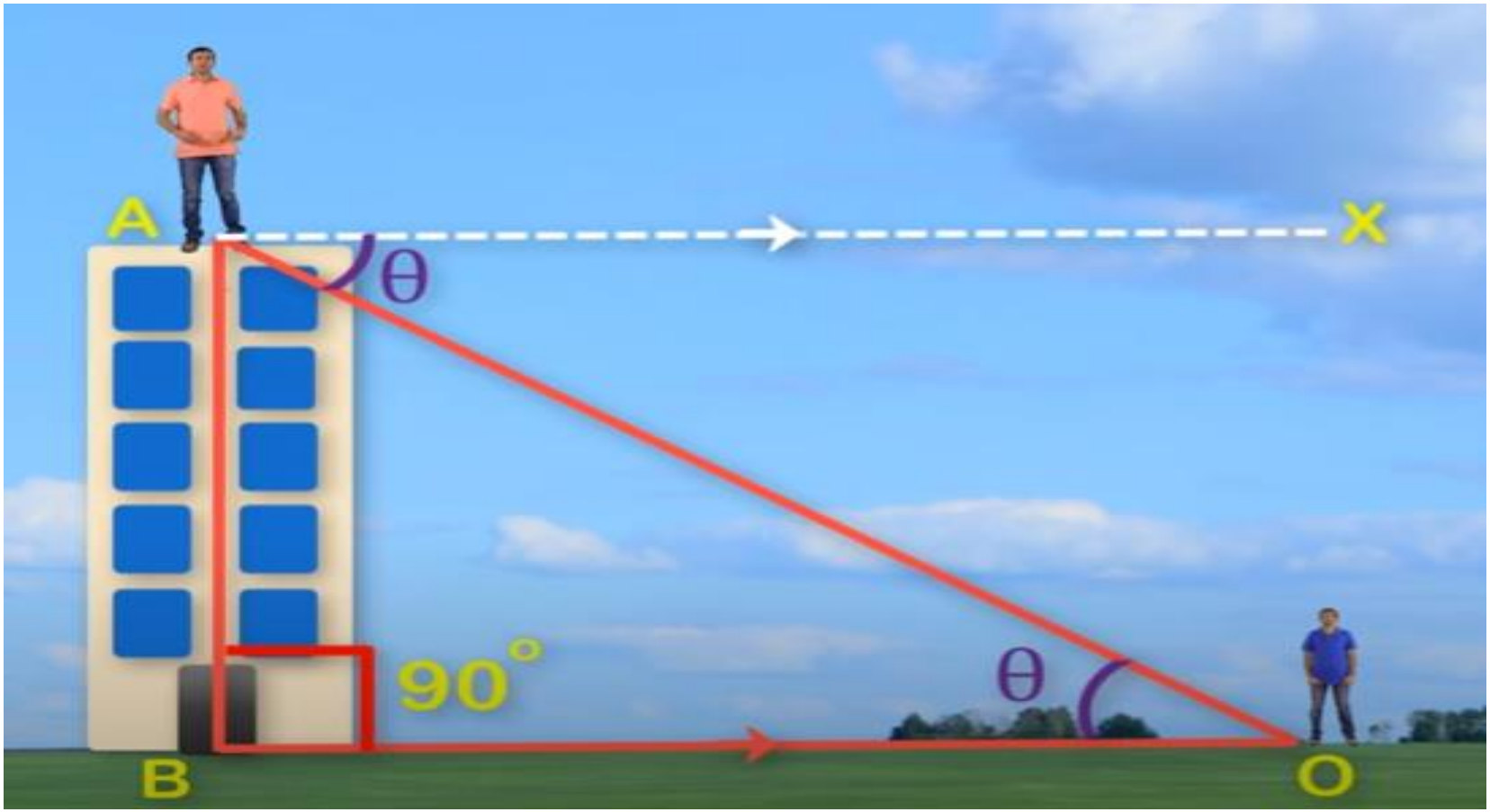
WHEN YOU OBSERVE A PLANE WHAT ANGLE DO YOU GET?

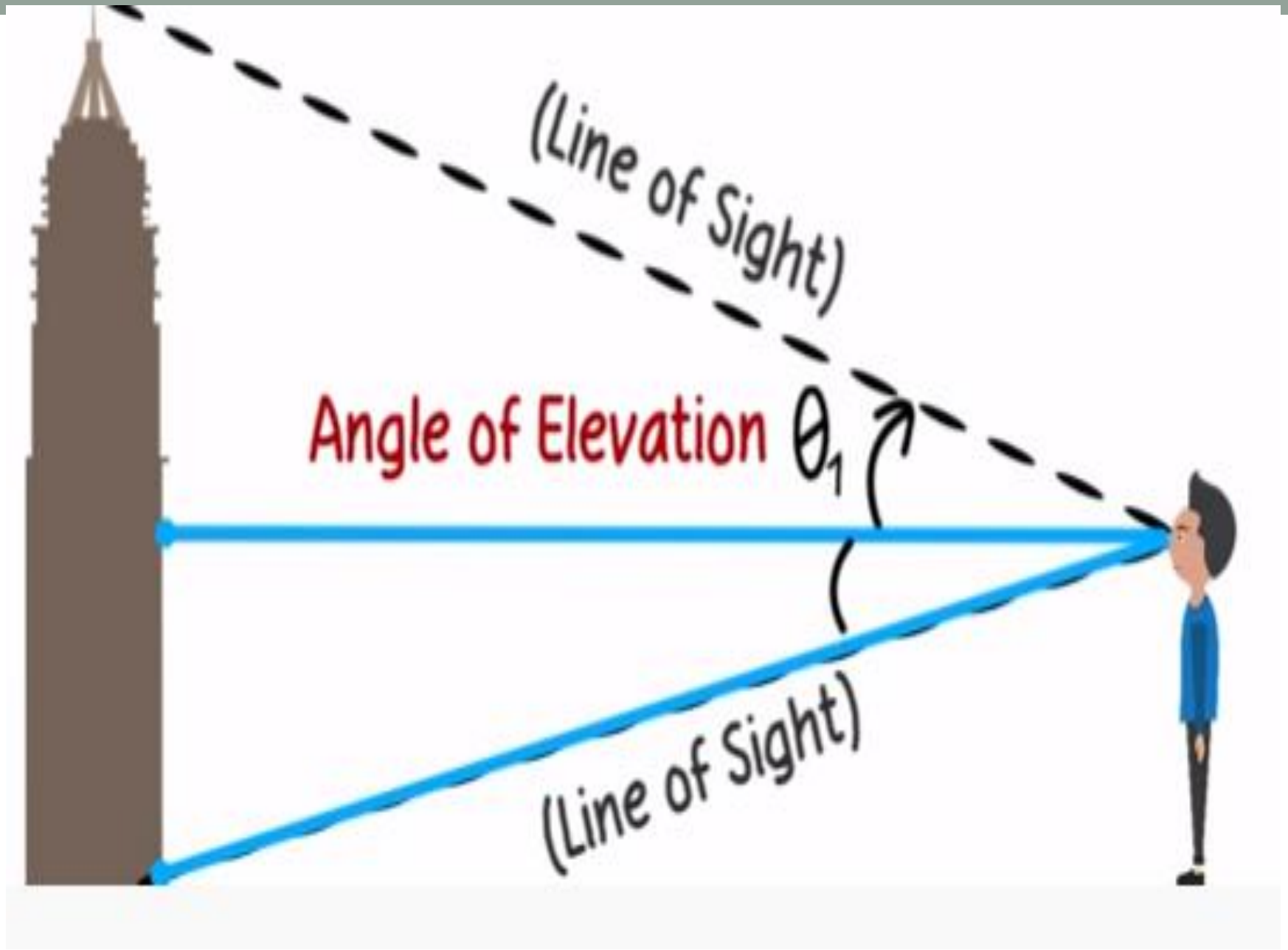


WHEN YOU OBSERVE A SMALL BUILDING FROM TOP OF A HILL WHAT DO YOU GET?


angle of depression
angle of elevation

The angle of depression of the person at point A becomes the angle of elevation of the person at point O ,since the horizontal lines are parallel to each other and the alternate interior angles formed are equal.




(Line of Sight)
Angle of Elevation θ_1
(Line of Sight)

Line of Sight → A line from an observer's eye to a distant point

Angle of Elevation → An angle between the line of sight and horizontal when the line of sight is ABOVE the horizontal

Angle of Depression → An angle between the line of sight and horizontal when the line of sight is BELOW the horizontal

The lamp post forms a shadow on the horizontal surface as shown in figure. The angle of elevation of the top of the lamp post from the tip of the shadow is 45° . The top of the lamp post is 20m away from the tip of the shadow. Find the height of the lamp post.

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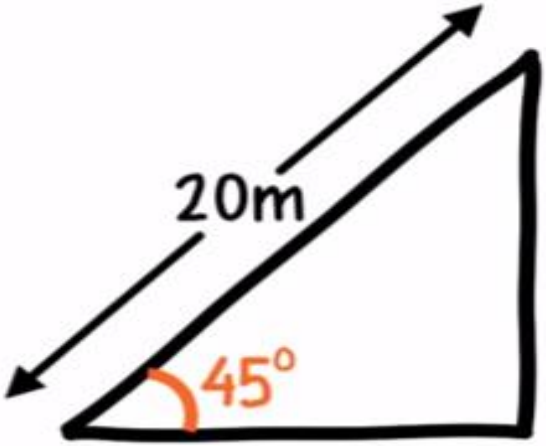
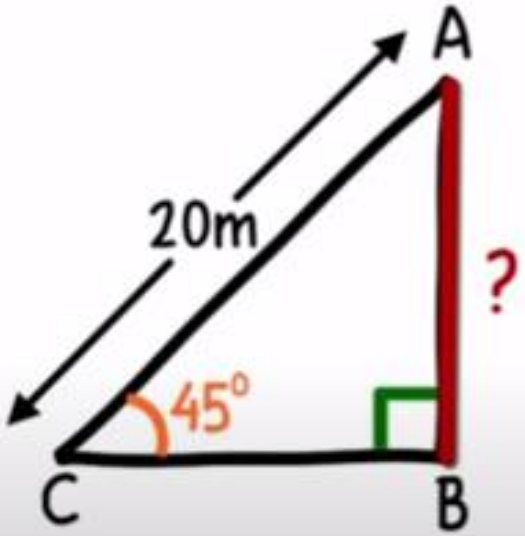


LET US CONVERT THE SITUATION TO A SIMPLE RIGHT TRIANGLE.

THE DISTANCE FROM THE TIP OF THE SHADOW TO THE TOP OF THE POLE IS 20M AND THE HT OF THE POLE IS ASKED. WHICH TWO SIDES ARE INVOLVED? THE OPPOSITE SIDE AND THE HYPOTENUSE. WHICH TRIGONOMETRIC RATIOS INVOLVE THE OPPOSITE AND THE HYPOTENUSE?



$$\sin = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\text{cosec} = \frac{\text{Hypotenuse}}{\text{Opposite}}$$

$$\sin C = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC}$$

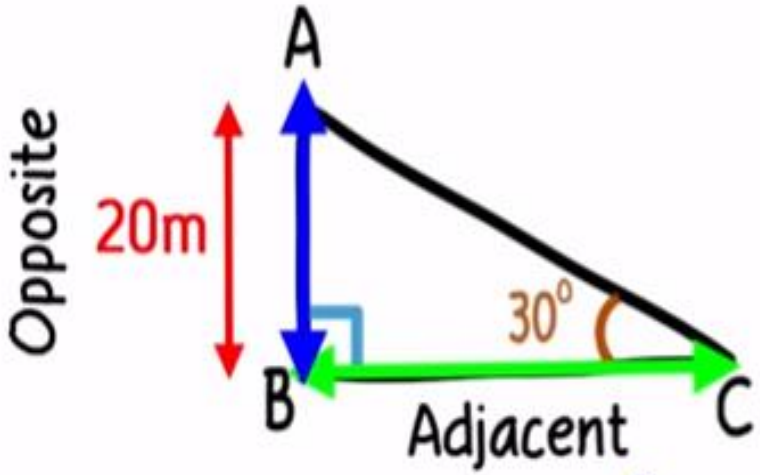
$$\sin 45^\circ = \frac{AB}{20}$$

$$\frac{1}{\sqrt{2}} = \frac{AB}{20}$$

$$AB = \frac{20}{\sqrt{2}}$$

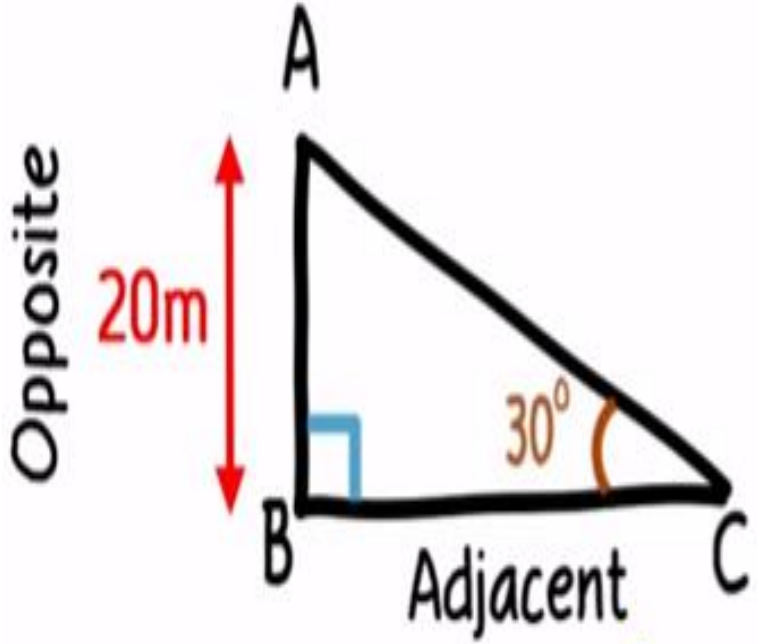
$$= \frac{20}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 10\sqrt{2}\ m$$

A ladder is leaning against a wall of height 20m. The angle made by the ladder with the horizontal ground surface is 30° . Find the distance from the foot of the ladder to the bottom of the wall.



$\angle C = 30^{\circ}$

$\ell\,(AB) = 20$ m



$\angle C = 30^{\circ}$

$\ell\,(AB) = 20\ m$

$$\tan C = \frac{AB}{BC}$$

$$\tan 30^{\circ} = \frac{20}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{20}{BC}$$

$$BC = 20\sqrt{3}\ m$$

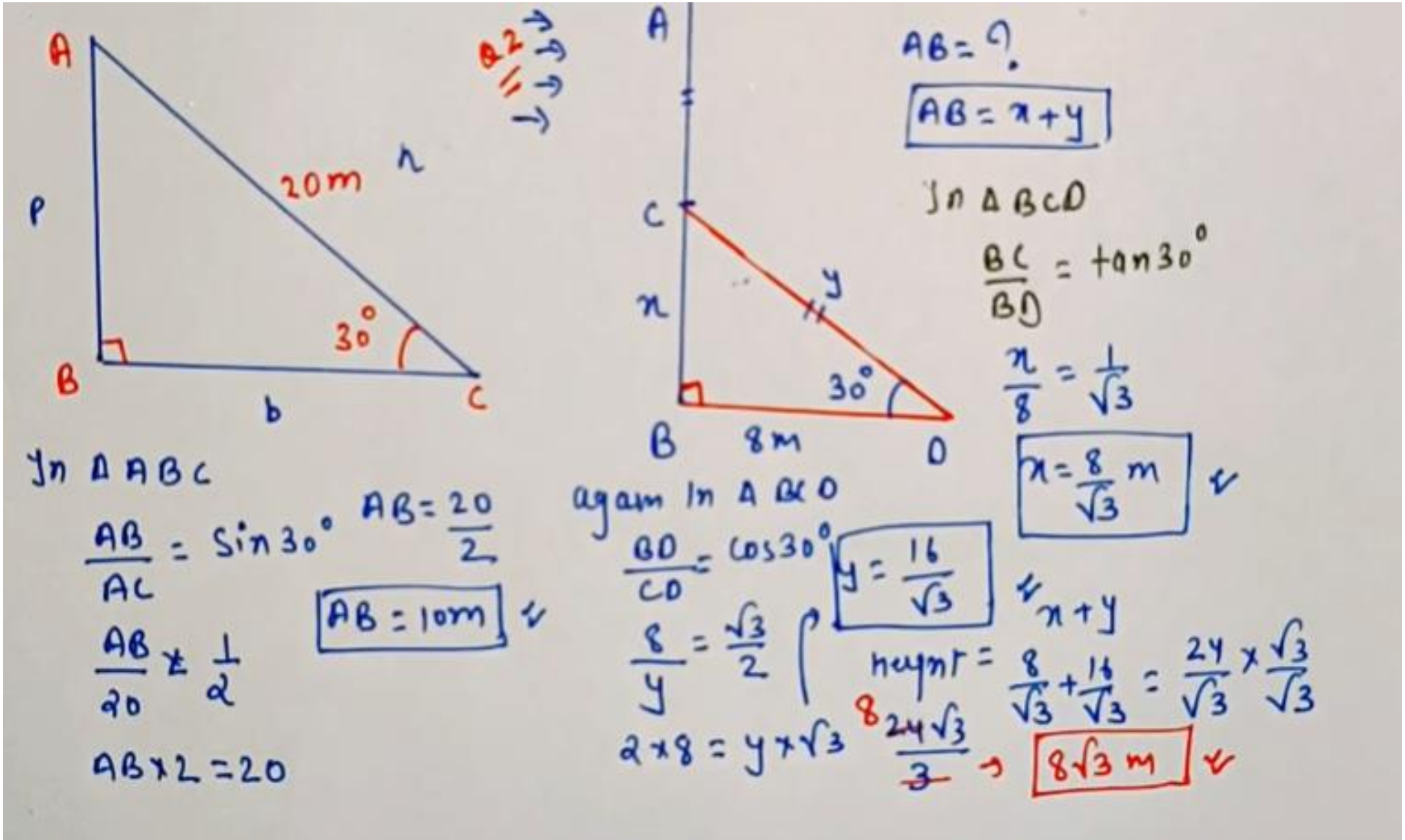
Let us solve a few text book sums.

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the pole is 30°.

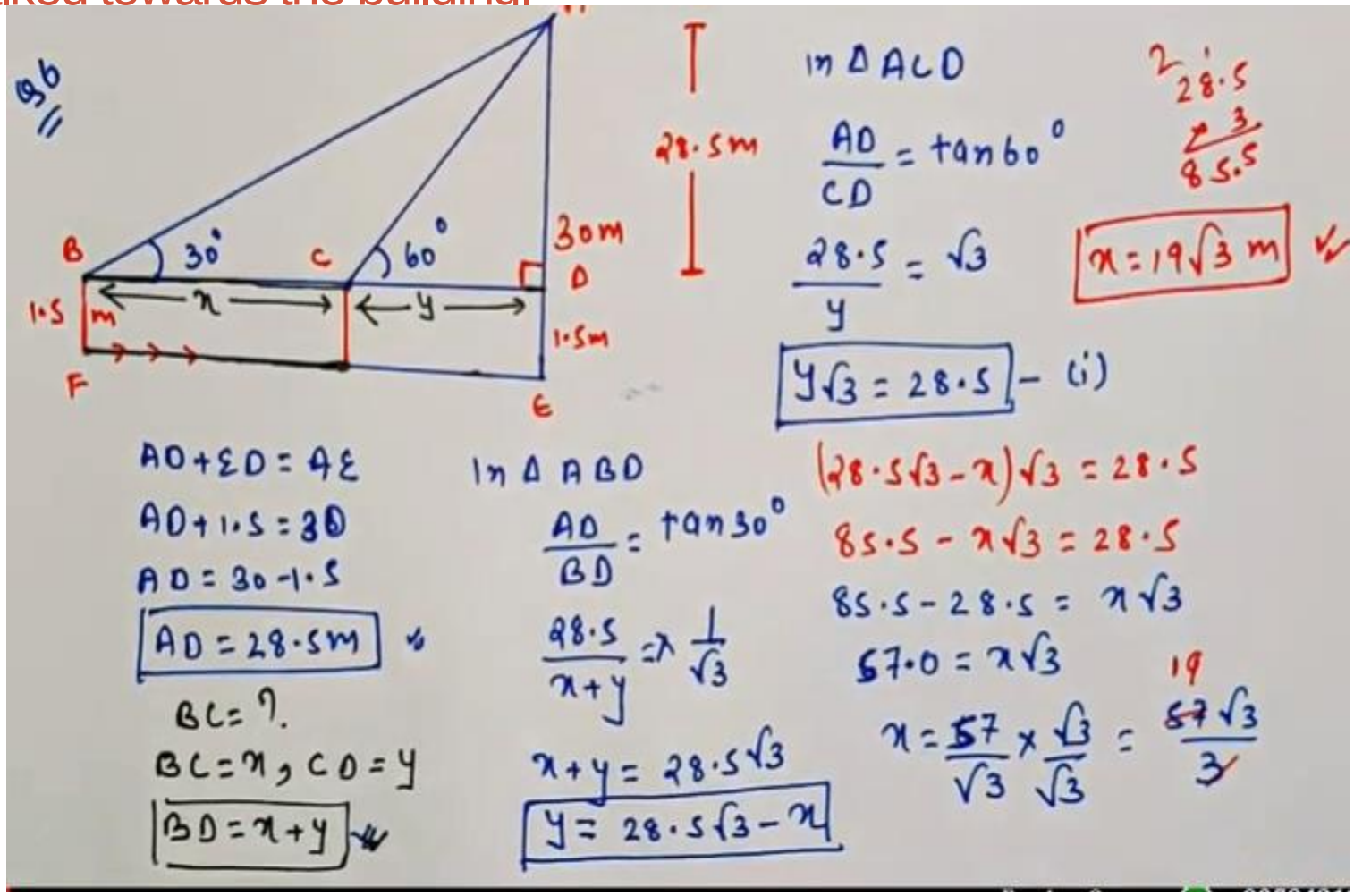
2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30 degrees with it. The point where the top touches the ground is 8m . Find the height of the tree.

Let the pole be AB and the rope be AC. We have to find AB.

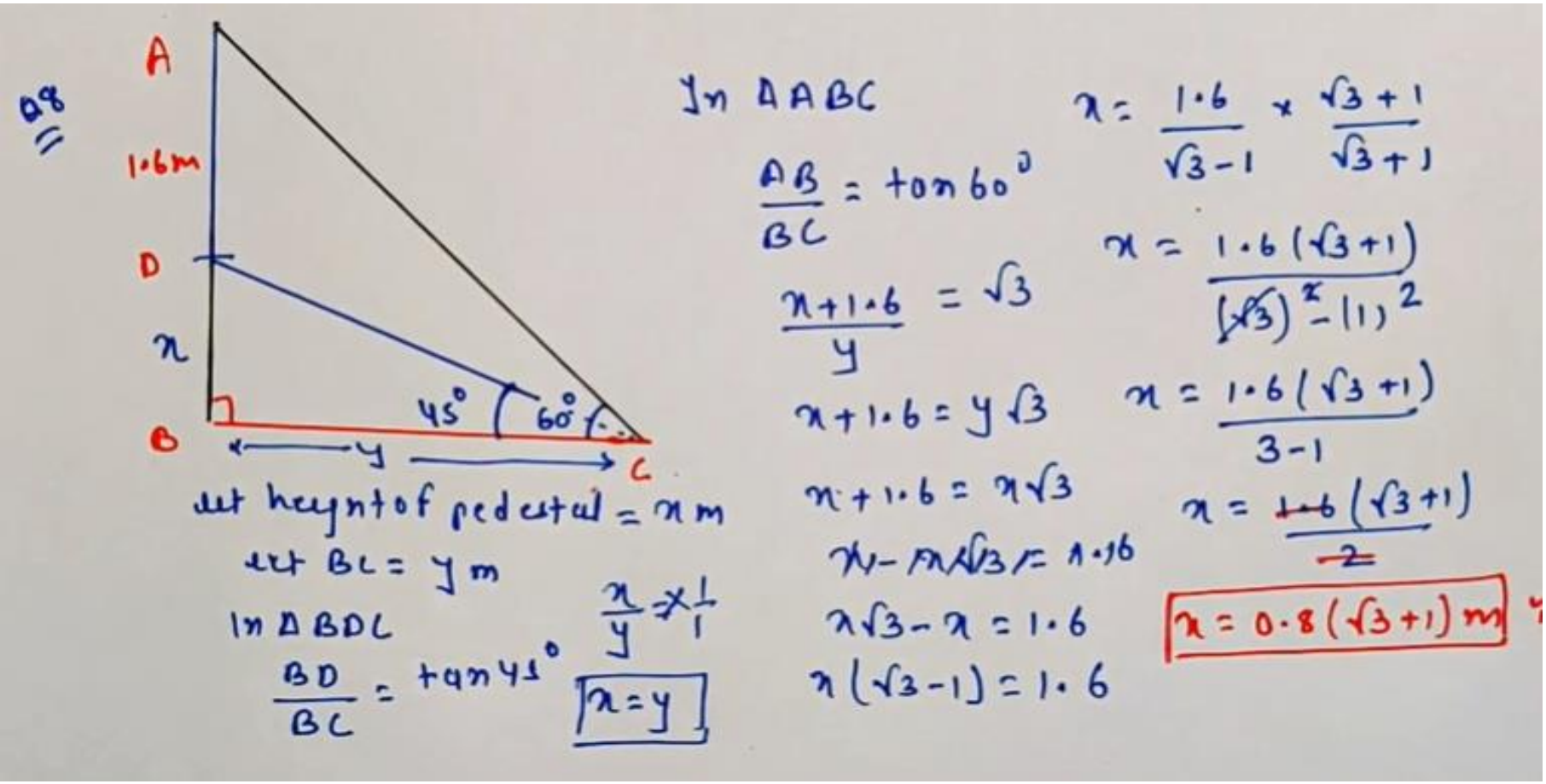
Let the tree be AB. The tree breaks at C and top of the tree reaches D.



A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.



A statue 1.6 m tall, stands on the top of a pedastal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.



Let height of pedestal = x m

Let BC = y m

In $\triangle BDC$

$\frac{BD}{BC} = \tan 45^\circ$

$\frac{x}{y} = \frac{1}{1}$

$\boxed{x = y}$

In $\triangle ABC$

$\frac{AB}{BC} = \tan 60^\circ$

$\frac{x+1.6}{y} = \sqrt{3}$

$x + 1.6 = y\sqrt{3}$

$x + 1.6 = x\sqrt{3}$

$x - x\sqrt{3} = -1.6$

$x\sqrt{3} - x = 1.6$

$x(\sqrt{3}-1) = 1.6$

$x = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$x = \frac{1.6(\sqrt{3}+1)}{(\sqrt{3})^2 - (1)^2}$

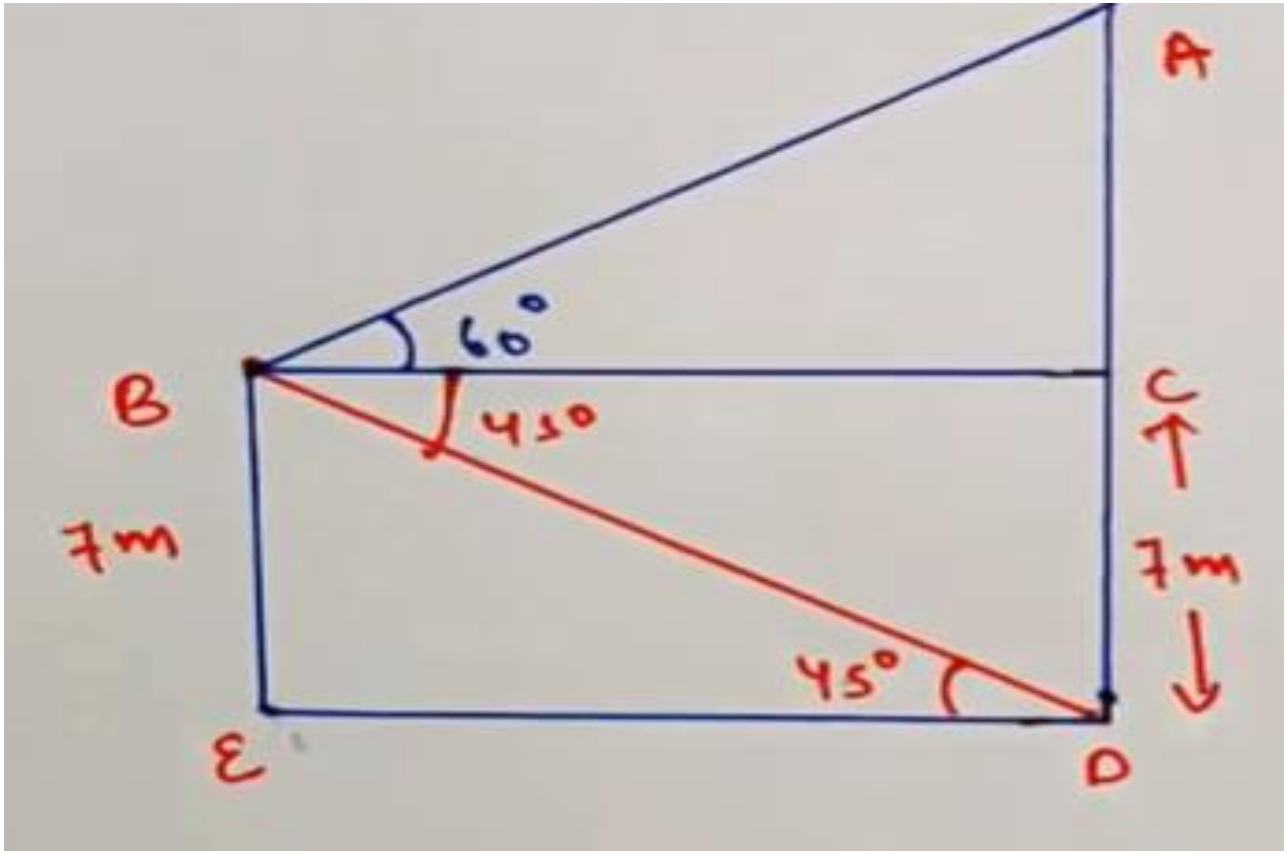
$x = \frac{1.6(\sqrt{3}+1)}{3-1}$

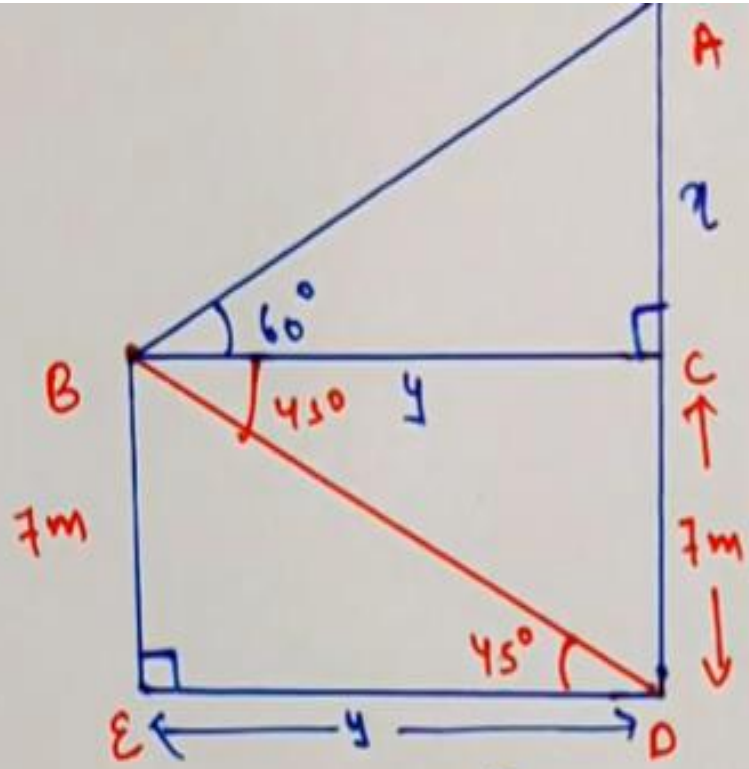
$x = \frac{1.6(\sqrt{3}+1)}{2}$

$\boxed{x = 0.8(\sqrt{3}+1)\ m}$

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution: Let us first draw a suitable figure . Let BE be the 7 m tall building and AD be the cable tower. Mark angle of elevation and the angle of depression.





Height of tower = ?

AD = ?

$AD = x + 7 = 7\sqrt{3} + 7$

$\boxed{7(\sqrt{3}+1)\ m}$ ✓

In $\triangle ABED$

$\frac{BE}{ED} = \tan 45^\circ$

$\frac{7}{y} \times \frac{1}{1}$

$\boxed{y = 7m}$

Now, in $\triangle ABE$

$\frac{AC}{BC} = \tan 60^\circ$

$\frac{x}{y} = \times \frac{\sqrt{3}}{1}$

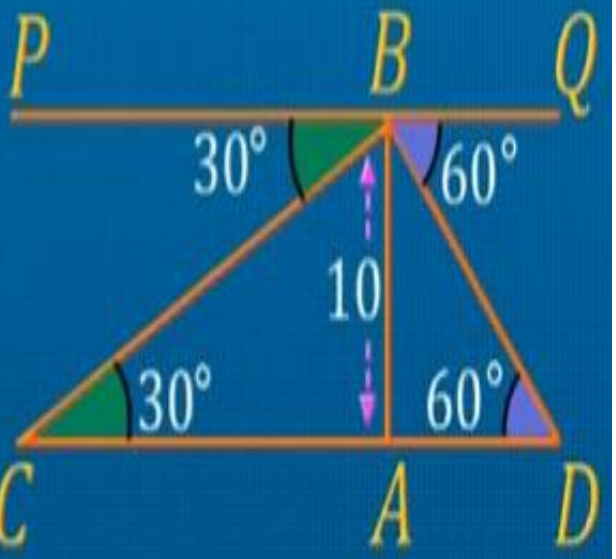
$x = y\sqrt{3}$

$\boxed{x = 7\sqrt{3}\ m}$

2 From a point on a bridge across a river, the angles of depression of two points on the banks on opposite sides of the river are 30° and 60° respectively. If the bridge is at a height of 10 m from the banks, find the width of the river.

Height of bridge $AB = 10\ m$

Angles of depression of banks C and D are 30° and 60° respectively.



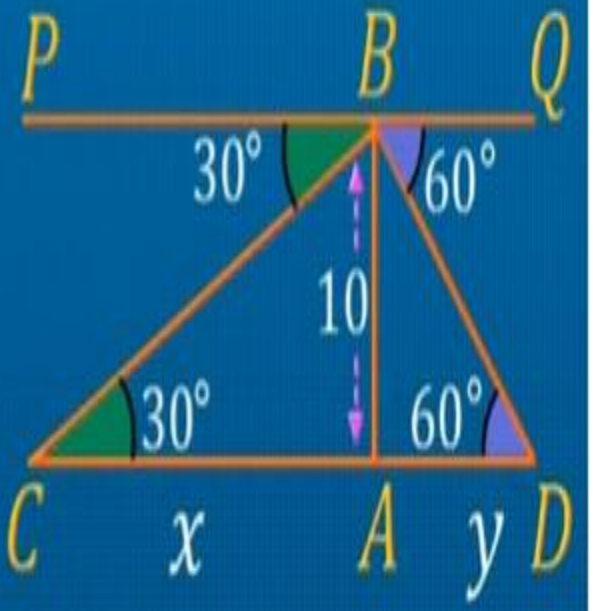
Let $AC = x$ and $AD = y$

In ΔBAC, $\angle ACB = 30°$

$$\tan 30° = \frac{AB}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{x}$$

$$x = 10\sqrt{3}$$

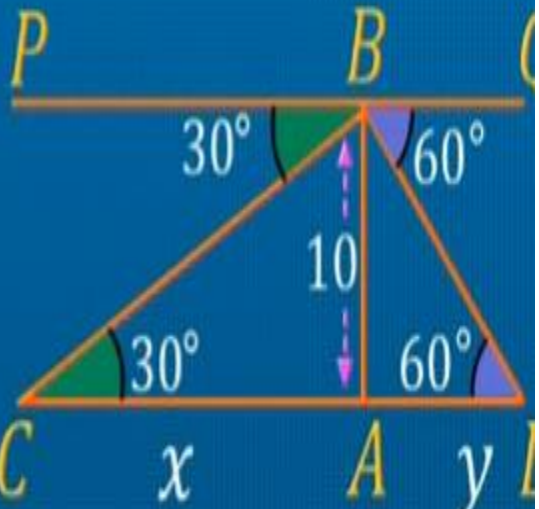


Let $AC = x$ and $AD = y$

$$x = 10\sqrt{3}$$

In ΔBAD,

$\angle ADB = 60^\circ$



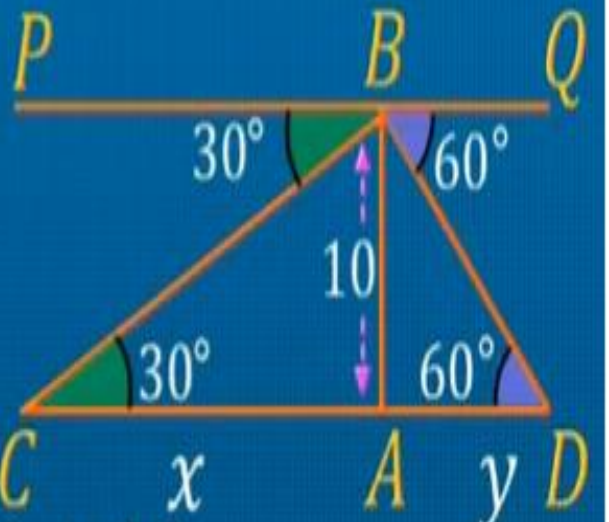
$$\tan 60^\circ = \frac{AB}{AD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{10}{y}$$

Let $AC = x$ and $AD = y$

$$x = 10\sqrt{3}$$

$$y = \frac{10}{\sqrt{3}}$$



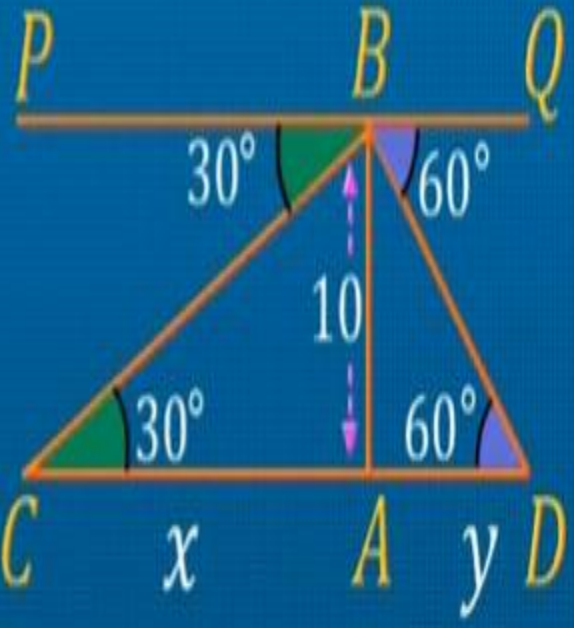
Width of the river $= x + y$

$$= 10\sqrt{3} + \frac{10}{\sqrt{3}} = \frac{30 + 10}{\sqrt{3}}$$

Width of the river $= x + y$

$$= \frac{30 + 10}{\sqrt{3}}$$

$$= \frac{40}{\sqrt{3}} = 23.1$$

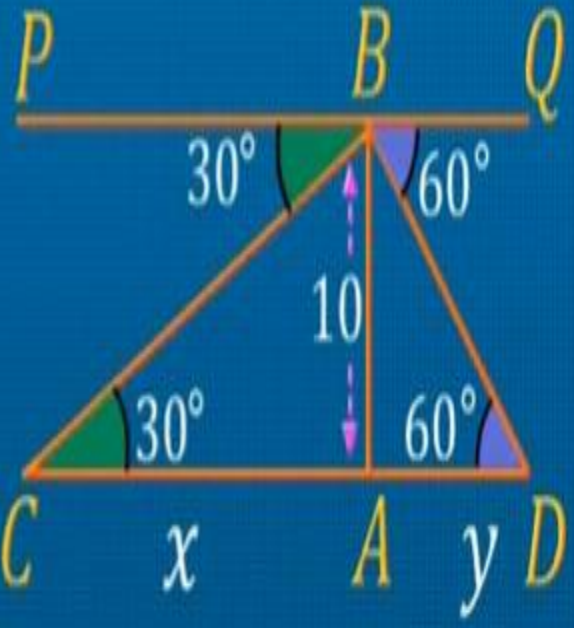


Therefore, the width of the river is 23.1 m.

Width of the river $= x + y$

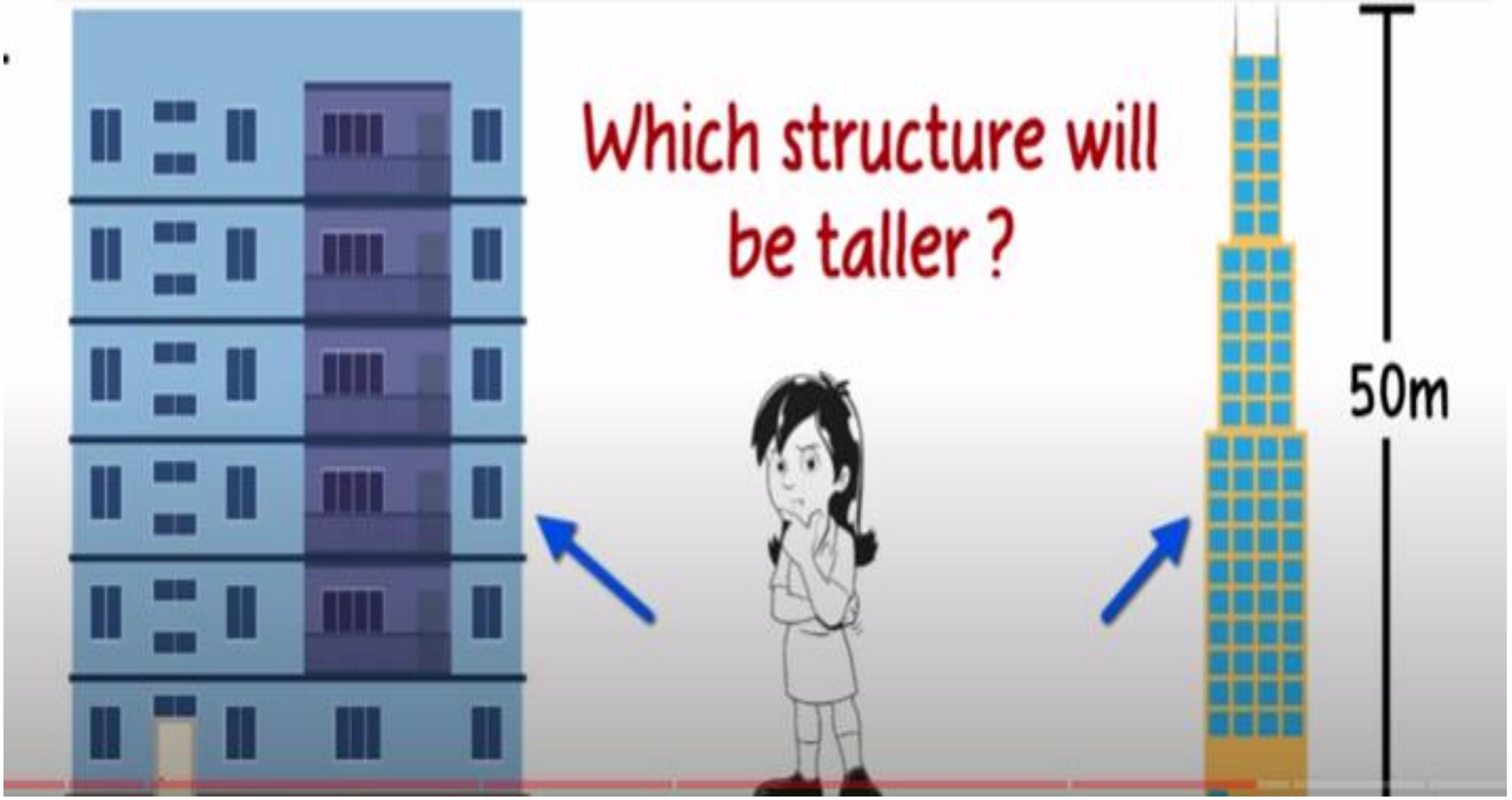
$$= \frac{30 + 10}{\sqrt{3}}$$

$$= \frac{40}{\sqrt{3}} = 23.1$$



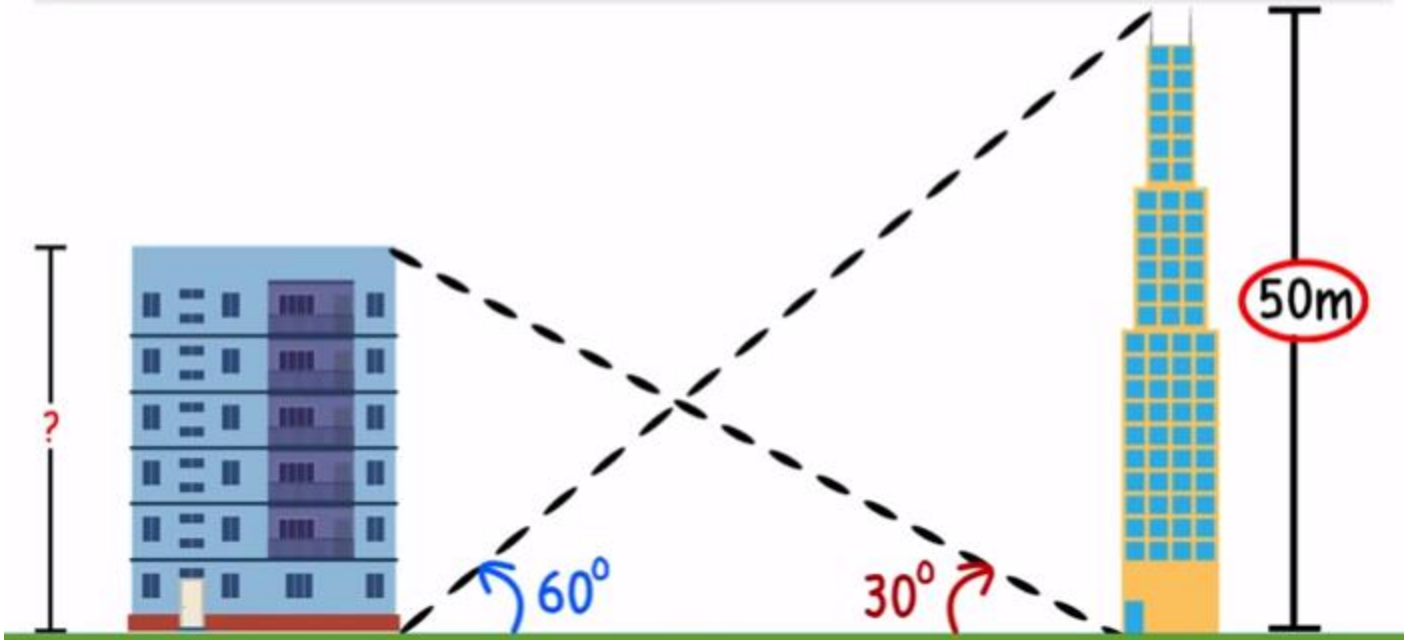
Therefore, the width of the river is 23.1 m.

The angle of elevation of the top of a building from the foot of the tower is 30°. and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50m high, find the height of the building.



You will lift your head more if the building is taller, right.! Therefore, whichever has a greater angle of elevation will be taller.

The angle of elevation of the top of a building from the foot of the tower is 30°. and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50m high, find the height of the building.



NOW YOU ARE IN A POSITION TO DO THE SUMS OF EXERCISE 9.1 IN YOUR NOTEBOOK.

ACTIVITY: MAKE A CLINOMETER, AN APPARATUS USED TO MEASURE THE ANGLES OF ELEVATION AND DEPRESSION.

THANK YOU!!!!!!!