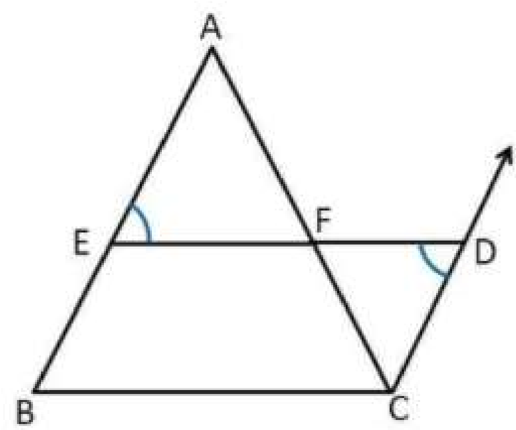
CHAPTER-8
QUADRILATERALS

HANDOUT-MODULE-3

MID-POINT THEOREM

Theorem 8.9 : The line segment joining the mid-points of two sides of a triangle is parallel to the third side and half of the third side.

GIVEN : ΔABC in which line segment EF joins the mid points E and F of AB and AC respectively.



TO PROVE : EF ∥ BC & EF = ½ BC

CONSTRUCTION : Through point C , draw CX ∥ AB . Extend EF to intersect CX at D

PROOF :
In ΔAFE & ΔCFD

∠AFE = ∠CFD[vertically opposite angles]
AF = CF(given)
∠FAE = ∠FCD [Alt. int. angles]

ΔAFE ≅ ΔCFD (ASA Rule)
EF = DF (cpct)----------(1)

AE = CD (cpct)---------(2)
But AE = BE (given)------(3)

From (2) and (3) we get
BE = CD
BE ∥ CD
Since one pair of sides are parallel and equal , quadrilateral BCDE is a parallelogram.
∴ ED ∥ BC(opp. Sides of a ∥gm)
i.e. EF ∥ BC
Hence proved
ED = BC (opp. Sides of a ∥gm)(4)
EF = DF = ½ ED (from 1)
EF = ½ BC [from (4)]
Hence proved

CONVERSE OF MID-POINT THEOREM

THEOREM 8.10 : The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

GIVEN : ΔABC in which E is the mid point of AB. A line l through point E parallel to BC intersects AC at F.

TO PROVE : F is the mid point of AC i.e. AF = CF

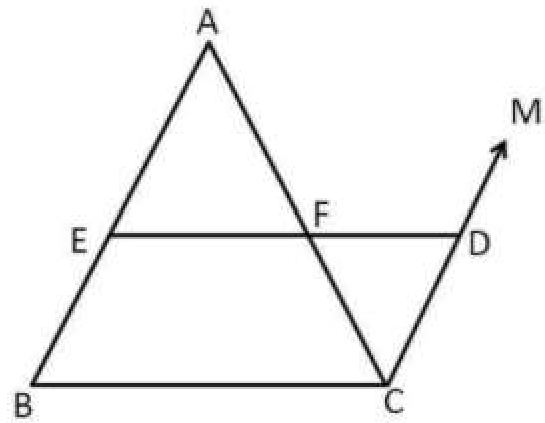
CONSTRUCTION : Through point C draw CX ∥ AB intersecting line l at point D

PROOF:

ED ∥ BC (given)

BE ∥ CD (by construction)

∴ quadrilateral BCDE is a ∥gm



BE=CD(opp.sides of a ∥gm)---(1)

BE = AE (given)-------(2)

From (1) and (2) we get

AE = CD ---------------------------(3)

In ΔAFE & ΔCFD

By alternate interior angles property

∠AEF = ∠CDF

∠FAE = ∠FCD

AE = CD (from 3)

ΔAFE ≅ ΔCFD (ASA Rule)

AF = CF (cpct)

∴ F is the mid point of AC

NOTE:-

The quadrilateral formed by joining the mid-points of the sides of a quadrilateral, in order, is a parallelogram.