

Handout

Module 1

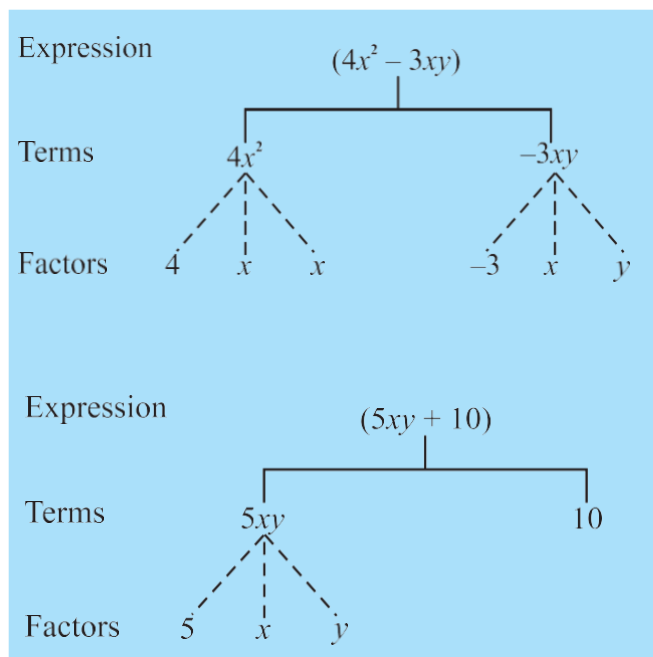
CHAPTER 12. ALGEBRAIC EXPRESSIONS

1. INTRODUCTION:

ALGEBRAIC Expressions are a central concept in algebra. A **variable** can take various values. Its value is not fixed. On the other hand, a **constant** has a fixed value. We combine variables and constants to make algebraic expressions. For this, we use the operations of addition, subtraction, multiplication and division. We have already come across expressions like $3x + 8$, $7y - 2$. The expression $3x + 8$ is obtained from the variable x , first by multiplying x by the constant 3 and then adding the constant 8 to the product. Similarly, $7y - 2$ is obtained by first multiplying y by 7 and then subtracting 2 from the product.

2. TERMS AND FACTORS OF AN EXPRESSION:

Algebraic expressions have parts which are formed separately and then added. Such parts of an expression which are formed separately first and then added are known as **terms**.



We saw above that the expression $(4x^2 - 3xy)$ consists of two terms $4x^2$ and $-3xy$. The term $4x^2$ is a product of 4, x and x ; we say that 4, x and x are the factors of the term $4x^2$. A term is a product of its factors. The term $-3xy$ is a product of the factors -3, x and y .

3. Coefficients

We have learnt how to write a term as a product of factors. One of these factors

may be numerical and the others algebraic (i.e., they contain variables). The numerical factor is said to be the numerical coefficient or simply the **coefficient** of the term. Thus in $5xy$, 5 is the coefficient of the term. It is also the coefficient of xy . In the term $10xyz$, 10 is the coefficient of xyz , in the term $-7x^2y^2$, -7 is the coefficient of x^2y^2 .

Sometimes, the word 'coefficient' is used in a more general way. Thus we say that in the term $5xy$, 5 is the coefficient of xy , x is the coefficient of $5y$ and y is the coefficient of $5x$. In $10xy^2$, 10 is the coefficient of xy^2 , x is the coefficient of $10y^2$ and y^2 is the coefficient of $10x$. Thus, in this more general way, a coefficient may be either a numerical factor or an algebraic factor or a product of two or more factors. It is said to be the coefficient of the product of the remaining factors.

4. LIKE AND UNLIKE TERMS:

When terms have the same algebraic factors, they are **like** terms. When terms have different algebraic factors, they are **unlike** terms. For example, in the expression $2xy - 3x + 5xy - 4$, look at the terms $2xy$ and $5xy$. The factors of $2xy$ are 2, x and y . The factors of $5xy$ are 5, x and y . Thus their algebraic (i.e., those which contain variables) factors are the same and hence they are **like** terms. On the other hand the terms $2xy$ and $-3x$, have different algebraic factors. They are **unlike** terms. Similarly, the terms, $2xy$ and 4, are unlike terms. Also, the terms $-3x$ and 4 are unlike terms.

By following simple steps you can decide whether the given terms are like or unlike terms:

- (i) *Ignore the numerical coefficients. Concentrate on the algebraic part of the terms.*
- (ii) *Check the variables in the terms. They must be the same.*
- (iii) *Next, check the powers of each variable in the terms. They must be the same.*

Note that in deciding like terms, two things do not matter (1) the numerical coefficients of the terms and (2) the order in which the variables are multiplied in the terms.

5. TYPES OF ALGEBRAIC EXPRESSIONS:

Types of algebraic expressions are MONOMIALS, BINOMIALS, TRINOMIALS AND POLYNOMIALS.

An expression with only one term is called a **monomial**; for example, $7xy$, $-5m$, $3x^5$ etc.

An expression which contains two unlike terms is called a **binomial**; for example, $x + y$, $m - 5$, $mn + 4m$, $a^2 - b^2$ are binomials. The expression $10pq$ is not a binomial; it is a monomial. The expression $(a + b + 5)$ is not a binomial. It contains three terms.

An expression which contains three terms is called a **trinomial**; for example, the expressions $x + y + 7$, $ab + a + b$, $3x^2 - 5x + 2$, $m + n + 10$ are trinomials. The expression $ab + a + b + 5$ is, however not a trinomial; it contains four terms and not three. The expression $x + y + 5x$ is not a trinomial as the terms x and $5x$ are like terms.

In general, an expression with one or more terms is called a **polynomial**. Thus a monomial, a binomial and a trinomial are all polynomials.

6. ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS:

Adding and subtracting like terms

The simplest expressions are monomials. They consist of only one term. To begin with we shall learn how to add or subtract like terms.

- Let us add $3x$ and $4x$. We know x is a number and so also are $3x$ and $4x$. Now,

$$3x + 4x = (3 \times x) + (4 \times x)$$

$$= (3 + 4) \times x \text{ (using distributive law)}$$

$$= 7 \times x = 7x$$

or $3x + 4x = 7x$

- Let us next add $8xy$, $4xy$ and $2xy$

$$8xy + 4xy + 2xy = (8 \times xy) + (4 \times xy) + (2 \times xy)$$

$$= (8 + 4 + 2) \times xy$$

$$= 14 \times xy = 14xy$$

- In the same way, subtract $5ab$ from $11ab$.

$$11ab - 5ab = (11 - 5) ab = 6ab$$

Thus, the sum of two or more like terms is a like term with a numerical coefficient equal to the sum of the numerical coefficients of all the like terms.

Similarly, the difference between two like terms is a like term with a numerical coefficient equal to the difference between the numerical coefficients of the two like terms.

Note, unlike terms cannot be added or subtracted the way like terms are added or subtracted. We have already seen examples of this, when 5 is added to x , we write the result as $(x + 5)$. Observe that in $(x + 5)$ both the terms 5 and x are retained.

Similarly, if we add the unlike terms $3xy$ and 7 , the sum is $3xy + 7$. If we subtract 7 from $3xy$, the result is $3xy - 7$. We can see the process of subtraction in the following

example:

Subtract $24ab - 10b - 18a$ from $30ab + 12b + 14a$.

$$30ab + 12b + 14a - (24ab - 10b - 18a)$$

$$= 30ab + 12b + 14a - 24ab + 10b + 18a$$

$$= 30ab - 24ab + 12b + 10b + 14a + 18a$$

$$= 6ab + 22b + 32a$$

When we **add** two algebraic expressions, the like terms are added as given above; the **unlike** terms are **left as they are**. Thus, the sum of $4x^2 + 5x$ and $2x + 3$ is $4x^2 + 7x + 3$; the like terms $5x$ and $2x$ add to $7x$; the unlike terms $4x^2$ and 3 are left as they are.

