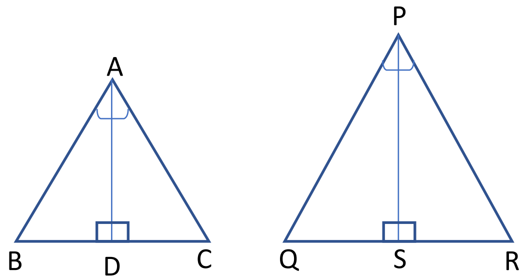
WORKSHEET ON MODULE 4/5 OF TRIANGLES

SOLVED EXAMPLE

1) Two isosceles triangles have equal vertical angles and their areas are in the ratio 9:16. Find the ratio of their corresponding heights (altitudes)



Given: ΔABC and ΔPQR are isosceles and $\angle BAC = \angle QPR$.
AD and PS are the altitudes.
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{16}$$

Solution:

In ΔABC andΔPQR, $AB = AC$ and $PQ = PR$ (isosceles triangles)

$$\Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$$

$\Rightarrow \Delta ABC \sim \Delta PQR$ (By SAS similarity condition)

We know that, the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} \qquad \text{---------} (1)$$

In ΔABD and ΔPQS, $\angle ADB = \angle PSQ = 90°$

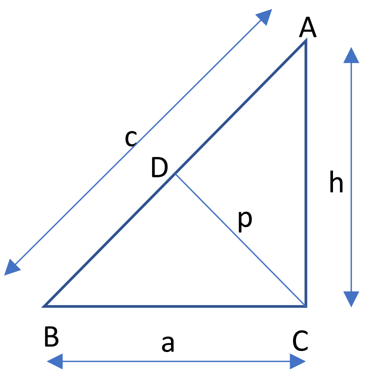
$\angle BAC = \angle QPR \Rightarrow \angle BAD = \angle QPS$

By AA similarity $\Delta ABD \sim \Delta PQS$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PS}$$

From (1), $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2} = \frac{9}{16} \Rightarrow \frac{AD}{PS} = \frac{3}{4}$

2) ABC is a right triangle right angled at C. Let BC = a, CA = b, AB = c and let p be the length of perpendicular from C on AB, prove that
   a. cp = ab
   b. $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$



Solution:

a. Draw $CD \perp AB$. Let CD = p

$ar(\Delta ABC) = \frac{1}{2}(BC \times CA) = \frac{1}{2}ab$ (Taking base as BC and altitude as AC) ------------ (1)

$ar(\Delta ABC) = \frac{1}{2}(AB \times CD) = \frac{1}{2}cp$ (Taking base as AB and altitude as CD) ------------ (2)

From (1) and (2), we get

cp = ab

b. We are given that ΔABC is a right-angled triangle with $\angle C = 90°$

Therefore, by Pythagoras theorem, we get

$AB^2 = BC^2 + AC^2$

$\Rightarrow c^2 = a^2 + b^2$

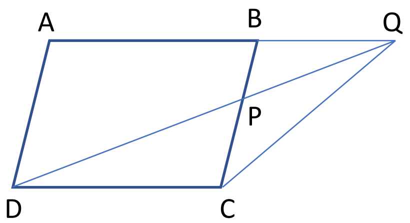
$\Rightarrow \left(\frac{ab}{p}\right)^2 = a^2 + b^2$ (since cp = ab)

$\Rightarrow \frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2}$

$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

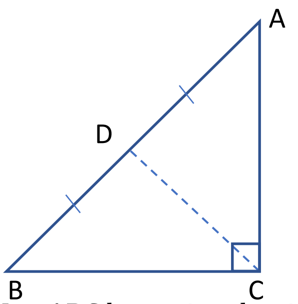
SOLVE THE FOLLOWING

1) In the given figure, ABCD is a parallelogram P is a point on BC, such that BP:PC = 1:2. DP is produced meets AB produced at Q. Given area of triangle BPQ = 20 cm², calculate the area of triangle DCP.



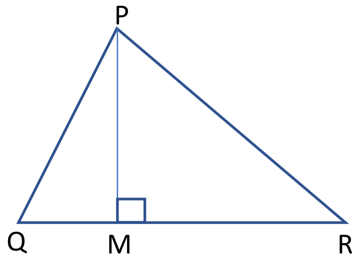
2) Prove that the ratio of areas of two similar triangles is equal to the squares of the ratios of their corresponding sides. Using the above theorem, given that the areas of two similar triangles are 81 cm² and 144 cm² and the largest side of the smaller triangle is 27 cm, find the largest side of the larger triangle.

3) ABC is a triangle, right angled at C and AC = √3 BC. Prove that $\angle ABC = 60°$



4) Let ABC be a triangle, right-angled at C. If D is the mid-point of BC, prove that $AB^2 = 4AD^2 - 3AC^2$

5) Prove that in a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. Use the above theorem and show that $PR^2 = PQ^2 + QR^2 - 2QM.QR$



6) A 6.5 m long ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall. Find the height of the wall where the top of the ladder touches it.
7) In the given figure, ΔABC is right-angled at C and DE ⊥ AB. Prove that ΔABC ~ ΔADE and hence find the lengths of AE and DE