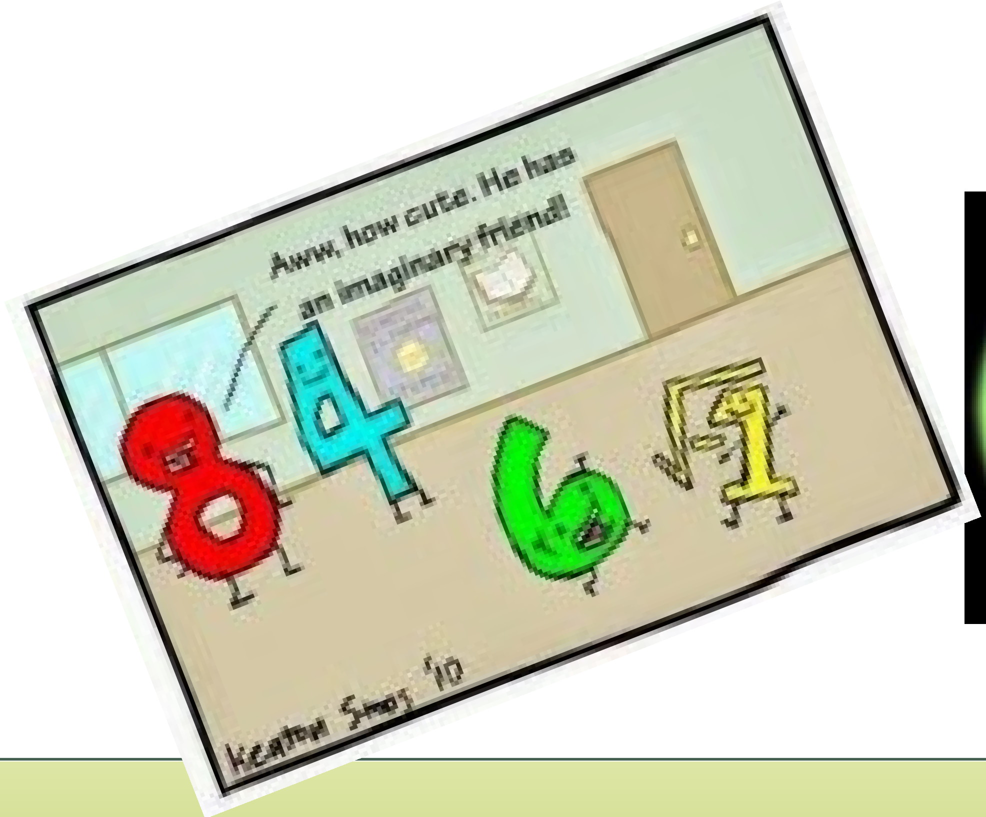
परमाणु ऊर्जा शिक्षण संस्था

(परमाणु ऊर्जा विभाग का स्वायत्त निकाय, भारत सरकार)

ATOMIC ENERGY EDUCATION SOCIETY

(An autonomous body under Department of Atomic Energy, Govt. of India)

5. Complex Numbers & Quadratic Equations



Module III

$$i^2=-1$$

e-content

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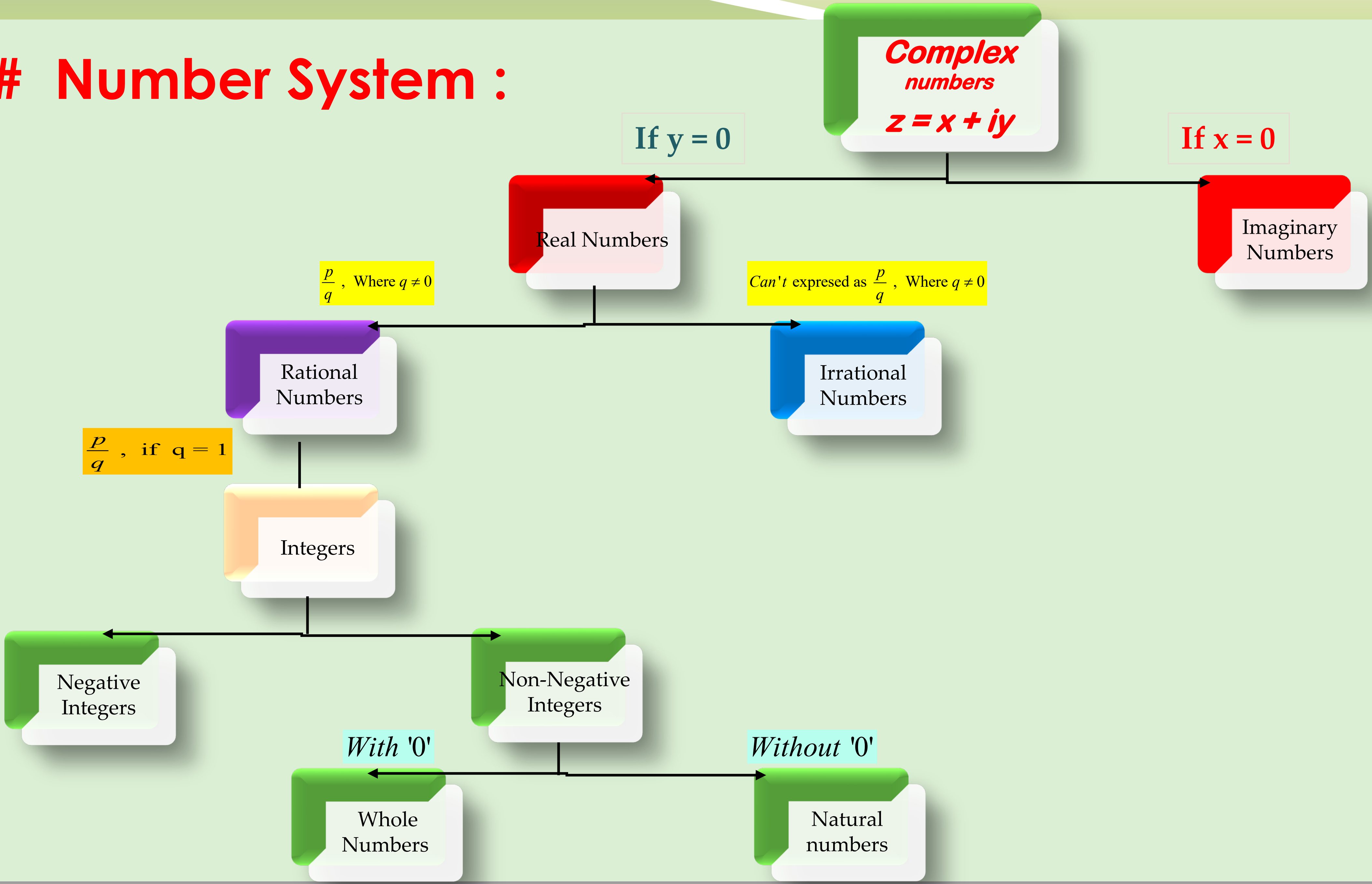
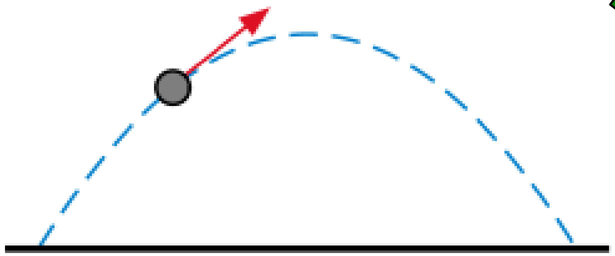
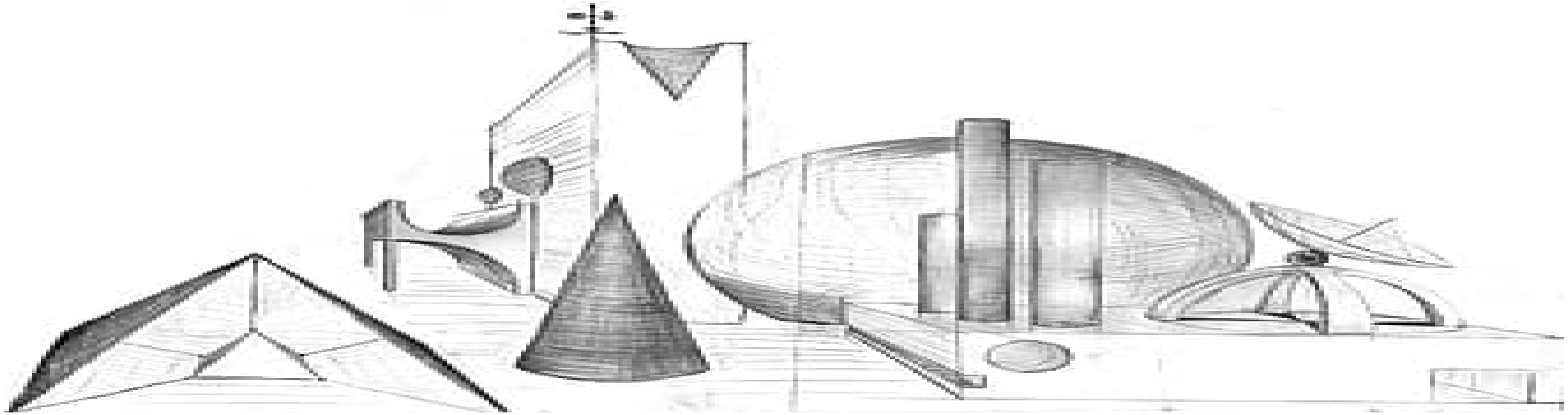
Number System :



Quadratic



Equations



- A quadratic equation is a second-order polynomial equation in a single variable x,

$$ax^2 + bx + c = 0, \qquad \text{where } a \neq 0$$

- The roots x can be found by completing the square, and we get the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

here, $D = b^2 - 4ac$ called discriminant.

We are already familiar with the quadratic equations and have solved them in the set of real numbers in the cases where discriminant is non-negative, i.e., $D \geq 0$

$ax^2 + bx + c = 0$ with real coefficients a, b, c and $a \neq 0$.

Also, let us assume that the $b^2 - 4ac < 0$.

Now, we know that we can find the square root of negative real numbers in the set of complex numbers. Therefore, the solutions to the above equation are available in the set of complex numbers which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{4ac - b^2}\, i}{2a}$$

Note At this point of time, some would be interested to know as to how many roots does an equation have? In this regard, the following theorem known as the *Fundamental theorem of Algebra* is stated below (without proof).

"A polynomial equation has at least one root."

As a consequence of this theorem, the following result, which is of immense importance, is arrived at:

"A polynomial equation of degree n has n roots."

Example 1 :

Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$

Solution Here, the discriminant of the equation is

$$1^2 - 4 \times \sqrt{5} \times \sqrt{5} = 1 - 20 = -19$$

Therefore, the solutions are

$$\frac{-1 \pm \sqrt{-19}}{2\sqrt{5}} = \frac{-1 \pm \sqrt{19}i}{2\sqrt{5}}.$$

Example 2 :

Solve $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

Solution : Re*write* the equation as

$$\sqrt{2}x^2 + x + \sqrt{2} = 0$$

here $a = \sqrt{2}$, $b = 1$ and $c = \sqrt{2}$

$$D = b^2 - 4ac = 1^2 - 4\left(\sqrt{2}\right)\left(\sqrt{2}\right)$$

$$= 1 - 8 = -7$$

Using Formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

we get, $x = \dfrac{-1 \pm \sqrt{-7}}{2\sqrt{2}} = \dfrac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$

A Question from JEE (Advanced) 2020

SECTION 1 (Maximum Marks: 18)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Q.1 Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of

$$ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$$

is

(A) 0 (B) 8000 (C) 8080 (D) 16000

Solution -

Here, a and b are real roots of $x^2 + 20x - 2020 = 0$

$$\therefore \quad a + b = -20, \qquad and\ ab = -2020$$

$$also,\ c\ and\ d\ are\ complex\ roots\ of\ x^2 - 20x + 2020 = 0$$

$$\therefore\ c + d = 20\ \ and\ cd = 2020$$

$$Now,\ simplifyng\ the\ given\ expression$$

$$ac(a-c) + ad(a-d) + bc(b-c) + bd(b-d)$$

$$= a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$$

$$= a^2c + a^2d + b^2c + b^2d - ac^2 - ad^2 - bc^2 - bd^2$$

$$= a^2(c+d) + b^2(c+d) - a(c^2+d^2) - b(c^2+d^2)$$

$$= (c+d)(a^2+b^2) - (a+b)(c^2+d^2)$$

$$We\ know\ that\ \ c + d = 20,\ cd = 2020\ \Rightarrow c^2 + d^2 = -3640$$

$$similarly\ \ a + b = -20,\ ab = -2020 \Rightarrow\ a^2 + b^2 = 4440$$

$$Puting\ these\ values\ we\ get$$

$$= (c+d)(a^2+b^2) - (a+b)(c^2+d^2)$$

$$= 20 \times 4440 - (-20) \times (-3640) = 20(800) = 16000$$

Hence option (D) is Correct.

Try These, Solve the following equations

1. $x^2 + 3x + 9 = 0$
2. $x^2 - x + 2 = 0$
3. $3x^2 - 4x + \frac{20}{3} = 0$
4. $27x^2 - 10x + 1 = 0$
5. $x^2 - 2x + \frac{3}{2} = 0$

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Complex Nos. Reference Book

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Chapter – 05 Complex Numbers (NCERT)

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