ATOMIC ENERGY CENTRAL SCHOOL, MYSORE

DISTANT LEARNING PROGRAM 2020-21

CLASS: XII – PHYSICS (CHAPTER-7)

ALTERNATING CURRENT

MODULE (2 of 2)

RESONANCE:

When the frequency of the applied alternating source(ω) is equal to the natural frequency of the RLC circuit, the current in the **circuit reaches its maximum value**. Then the circuit is said to be in **electrical resonance**. The frequency at which resonance takes place is called resonant frequency.

The current amplitude in an RLC circuit is given by

$$i_m = \frac{v_m}{Z}$$

(or)

$$i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}} \longrightarrow (1)$$

where, Z = $\sqrt{R^2 + (X_C - X_L)^2}$ is called 'impedance' of the circuit.

But, $X_C = \frac{1}{\omega C}$ and $X_L = \omega L$. The frequency ω of the applied AC voltage is varied continuously.

At a particular frequency $\omega = \omega_0$, called as resonant frequency

$$X_C = X_L \longrightarrow (2)$$

$$\frac{1}{\omega_0 C} = \omega_0 L$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow (3)$$

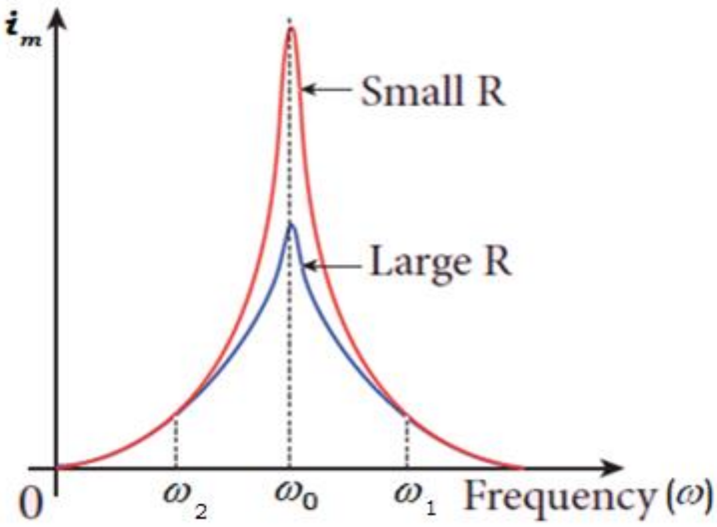
At resonance, **impedance in the circuit is minimum** and it is equal to the resistance R.

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + 0} = R$$

At resonance, **current in the circuit is maximum**. It is given by

$$i_m = \frac{v_m}{Z} = \frac{v_m}{R} \quad (\because Z = R) \longrightarrow (4)$$

The voltage across L and C cancel each other. The net voltage is equal to the voltage across



The maximum current at series resonance is limited by the resistance (R) of the circuit.

For smaller resistance, larger current with sharper curve is obtained and vice versa.

Applications of series RLC resonant circuit:

RLC circuits have many applications like filter circuits, oscillators, voltage multipliers etc. An important use of series RLC resonant circuits is in the tuning circuits of radio and TV systems.

NOTE:

It is important to note that resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit. Only then do the voltages across L and C cancel each other (both being out of phase) and the current amplitude is $\frac{v_m}{R}$, the total source voltage appearing across R. This means that **we cannot have resonance in a RL or RC circuit.**

SHARPNESS OF RESONANCE: (Assuming predominantly inductive circuit, ie; $X_L > X_C$

The amplitude of current in series LCR circuit is given by

$$i_m = \frac{v_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

(or)

$$i_m = \frac{v_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

The current is maximum when $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$. Its maximum value is given by

$$i_m^{max} = \frac{v_m}{R}$$

From the above resonance curve, it is found that the curve is symmetrical at $\omega = \omega_1$ and

$\omega = \omega_2$.

$$\omega_1 = \omega_0 + \Delta\,\omega$$

$$\omega_2 = \omega_0 - \Delta\,\omega$$

$$\omega_1 - \omega_2 = 2\,\Delta\,\omega = \text{Band width of the circuit.}$$

The quantity $\frac{\omega_0}{2\,\Delta\,\omega}$ is a measure of **'sharpness of resonance'.**

If $\Delta\,\omega$ is smaller, resonance is sharper.

Suppose we choose a value of ω for which the current amplitude is $\frac{1}{\sqrt{2}}$ times its maximum value.

At this value, the power dissipated by the circuit becomes half.

EXPRESSION FOR BAND WIDTH:

Current Amplitude $\quad i_m = \frac{1}{\sqrt{2}}\, i_m^{max} \longrightarrow$ (1)

For $\omega_1 = \omega_0 + \Delta\,\omega$, $\quad i_m = \frac{v_m}{\sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}} \longrightarrow$ (2)

Equating (1) and (2),

$$\frac{1}{\sqrt{2}}\, i_m^{max} = \frac{v_m}{\sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}}$$

$$\frac{1}{\sqrt{2}}\,\frac{v_m}{R} = \frac{v_m}{\sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}}$$

$$\frac{1}{\sqrt{2}\,R} = \frac{1}{\sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}}$$

$$\sqrt{2}\,R = \sqrt{R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2}$$

Squaring on both sides,

$$2\,R^2 = R^2 + (\omega_1 L - \frac{1}{\omega_1 C})^2$$

$$R^2 = (\omega_1 L - \frac{1}{\omega_1 C})^2$$

(or) $\quad R = \omega_1 L - \frac{1}{\omega_1 C} \longrightarrow$ (3)

Substituting $\omega_1 = \omega_0 + \Delta\,\omega$,

$$(\omega_0 + \Delta\,\omega)\,L - \frac{1}{(\omega_0 + \Delta\,\omega)\,C} = R$$

$$\omega_0 L\,(1 + \frac{\Delta\,\omega}{\omega_0}) - \frac{1}{\omega_0 C\,(1 + \frac{\Delta\,\omega}{\omega_0})} = R$$

Multiplying and dividing the second term on LHS by ω_0 L,

$$\omega_0 \mathrm{L} (1 + \frac{\Delta \omega}{\omega_0}) - \frac{\omega_0 \mathrm{L}}{(\omega_0 \mathrm{L}) \ \omega_0 \mathrm{C} (1 + \frac{\Delta \omega}{\omega_0})} = \mathrm{R}$$

$$\omega_0 \mathrm{L} (1 + \frac{\Delta \omega}{\omega_0}) - \frac{\omega_0 \mathrm{L}}{\omega_0{}^2 \mathrm{L} \mathrm{C} (1 + \frac{\Delta \omega}{\omega_0})} = \mathrm{R}$$

But, $\omega_0{}^2 = \frac{1}{\mathrm{LC}}$ and hence, $\omega_0{}^2 \mathrm{L} \mathrm{C} = 1$

$$\therefore \quad \omega_0 \mathrm{L} (1 + \frac{\Delta \omega}{\omega_0}) - \frac{\omega_0 \mathrm{L}}{(1 + \frac{\Delta \omega}{\omega_0})} = \mathrm{R}$$

(or) $\omega_0 \mathrm{L} (1 + \frac{\Delta \omega}{\omega_0}) - \omega_0 \mathrm{L} (1 + \frac{\Delta \omega}{\omega_0})^{-1} = \mathrm{R}$

Since $\frac{\Delta \omega}{\omega_0} \ll 1$, $(1 + \frac{\Delta \omega}{\omega_0})^{-1} = (1 - \frac{\Delta \omega}{\omega_0})$

$$\therefore \ \omega_0 \mathrm{L} (1 + \frac{\Delta \omega}{\omega_0}) - \omega_0 \mathrm{L} (1 - \frac{\Delta \omega}{\omega_0}) = \mathrm{R}$$

$$\cancel{\omega_0 \mathrm{L}} + \cancel{\omega_0} \mathrm{L} \left(\frac{\Delta \omega}{\cancel{\omega_0}} \right) - \cancel{\omega_0 \mathrm{L}} + \omega_0 \mathrm{L} \left(\frac{\Delta \omega}{\omega_0} \right) = \mathrm{R}$$

(or) $\mathrm{L} \Delta \omega + \mathrm{L} \Delta \omega = \mathrm{R}$

(or) $2 \mathrm{L} \Delta \omega = \mathrm{R}$

Hence, $$\boxed{\Delta \omega = \frac{\mathrm{R}}{2\mathrm{L}}} \longrightarrow (4)$$

<u>Sharpness of Resonance</u>:

It is given by $\frac{\omega_0}{2\Delta \omega} = \frac{\omega_0}{2 (\frac{R}{2L})} = \frac{\omega_0 \mathrm{L}}{R}$ = **Quality Factor** (Q)

$$\therefore \quad \boxed{Q = \frac{\omega_0 \mathrm{L}}{R}} \longrightarrow (5)$$

But, $\omega_0 = \frac{1}{\sqrt{\mathrm{LC}}}$

$$\therefore \quad Q = \frac{L}{R \sqrt{\mathrm{LC}}} = \frac{\sqrt{L^2}}{R \sqrt{\mathrm{LC}}}$$

(or) $$\boxed{Q = \frac{1}{R} \sqrt{\frac{L}{C}}} \longrightarrow (6)$$

Also, it can be shown that,

$$\boxed{Q = \frac{1}{\omega_0 CR}} \longrightarrow (7)$$

<u>NOTE</u>:

1. The physical meaning is that Q–factor indicates the number of times the voltage across L or C is greater than the applied voltage at resonance.

2. The selectivity or sharpness of a resonant circuit is measured by the quality factor or factor.

<u>POWER IN AC CIRCUIT</u>: (**THE POWER FACTOR**)

The instantaneous voltage applied to the series RLC circuit is

$$v = v_m \sin \omega t \longrightarrow (1)$$

The instantaneous current in the circuit is given by

$$i = i_m \sin(\omega t + \phi) \longrightarrow (2)$$

where, $\phi \rightarrow$ phase difference between the voltage across the source and current in the circuit.

It is given by $$\phi = \tan^{-1} \frac{(X_C - X_L)}{R} \longrightarrow (3)$$

The instantaneous power supplied by the source is equal to the product of the instantaneous current and instantaneous voltage.

$$p = vi = v_m \sin \omega t \times i_m \sin(\omega t + \phi)$$

$$= v_m \sin \omega t \times i_m [\sin \omega t . \cos \phi + \cos \omega t . \sin \phi]$$

(or) $$p = v_m i_m \times \sin \omega t [\sin \omega t . \cos \phi + \cos \omega t . \sin \phi]$$

$$= v_m i_m [\sin^2 \omega t . \cos \phi + (\sin \omega t . \cos \omega t) \sin \phi]$$

Taking average over one cycle of AC on both sides it is found that,

$$\langle \sin^2 \omega t \rangle = \frac{1}{2} \text{ and } \langle (\sin \omega t . \cos \omega t) \rangle = 0$$

$$\therefore \quad p_{av} = P = v_m i_m (\frac{1}{2} \cos \phi)$$

$$= \frac{v_m}{\sqrt{2}} \times \frac{i_m}{\sqrt{2}} \times \cos \phi$$

(or) $$\boxed{P = V_{rms} \times I_{rms} \times \cos \phi} \longrightarrow (4)$$

(or) $$\boxed{P = V I \cos \phi} \longrightarrow (5)$$

But, V = I Z. ∴ $$\boxed{P = I^2 Z \cos \phi} \longrightarrow (6)$$

So, the average power dissipated depends not only on the voltage and current but also on the cosine of the phase angle φ between them.

The term $\boldsymbol{\cos \phi}$ is called as **'power factor'.**

<u>SPECIAL CASES</u>:

<u>Case (i) Resistive circuit</u>: If the circuit contains only pure R, it is called resistive.

In that case, $\phi = 0$, $\cos \phi = 1$. There is maximum power dissipation.

Case (ii) Purely inductive or capacitive circuit: If the circuit contains only an inductor or capacitor, we know that the phase difference between voltage and current is $\phi = \frac{\pi}{2}$.

Therefore, $\cos\phi = \cos\frac{\pi}{2} = 0$, and no power is dissipated even though a current is flowing in the circuit. This current is sometimes referred to as **wattless current**.

Case (iii) LCR series circuit: In an LCR series circuit, power dissipated is given by the equation $P = I^2 Z \cos\phi$, where $\phi = \tan^{-1}\frac{(X_C - X_L)}{R}$. So, ϕ may be non-zero in a RL or RC or RCL circuit. Even in such cases, power is dissipated only in the resistor.

Case (iv) Power dissipated at resonance in LCR circuit:

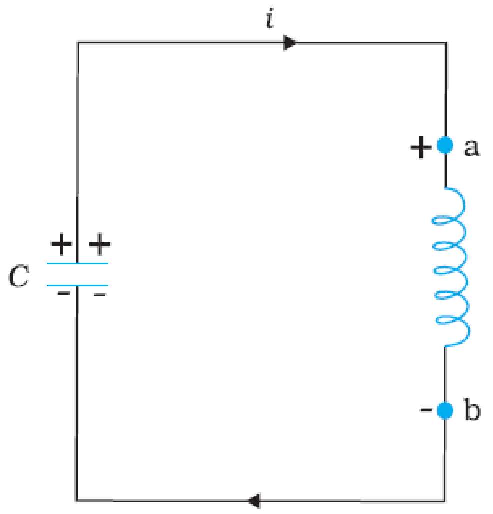
At resonance $X_C - X_L = 0$, and $\phi = 0$.

Therefore, $\cos\phi = 1$ and $P = I^2 Z = I^2 R$. That is, maximum power is dissipated in a circuit (through R) at resonance.

LC OSCILLATIONS:

In inductors, the energy is stored in the form of magnetic field while in capacitors; it is stored as the electric field.

Whenever energy is given to a circuit containing a pure inductor of inductance *L* and a capacitor of capacitance *C*, the energy oscillates back and forth between the magnetic field of the inductor and the electric field of the capacitor. Thus the electrical oscillations of definite frequency are generated. These oscillations are called *LC* oscillations.



Let a capacitor be charged q_m (at $t = 0$) and connected to an inductor as shown in the above figure. The moment the circuit is completed, the charge on the capacitor starts decreasing, giving rise to current in the circuit. Let *q* and *i* be the charge and current in the circuit at time *t*.

Since $\frac{di}{dt}$ is positive, the induced emf in *L* will have polarity as shown, i.e., $v_b < v_b$.

According to Kirchhoff's loop rule,

$$\frac{q}{C} - L\left(\frac{di}{dt}\right) = 0 \longrightarrow (1)$$

Since q decreases and i increases,

The current is given by $$i = -\frac{dq}{dt}$$

$$\frac{di}{dt} = -\frac{d}{dt}\left(-\frac{dq}{dt}\right) = \frac{d^2q}{dt^2}$$

Hence equn (1) becomes, $$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0$$

Dividing by L, $$\frac{q}{LC} + \frac{d^2q}{dt^2} = 0$$

(or) $$\frac{d^2q}{dt^2} + \left(\frac{1}{LC}\right)q = 0 \longrightarrow (2)$$

Equn (2) is a differential equation of Simple Harmonic Motion of the form

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0 \longrightarrow (3)$$

The charge q oscillates with a natural frequency given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow (4)$$

The charge q varies sinusoidally with time according to the equation

$$q = q_m \cos(\omega_0 t + \phi) \longrightarrow (5)$$

where, $q_m \rightarrow$ maximum value of q

and $\phi \rightarrow$ phase constant

At a time t = 0, $q = q_m$, $\cos\phi = 1$ and hence $\phi = 0$

If $\phi = 0$ in equn (5), then $$\boxed{q = q_m \cos\omega_0 t} \longrightarrow (6)$$

The current is given by $$i = -\frac{dq}{dt} = -\frac{d}{dt}(q_m \cos\omega_0 t)$$

$$= -q_m\left[\frac{d}{dt}(\cos\omega_0 t)\right]$$

$$= -q_m[-\sin\omega_0 t]\,\omega_0$$

(or) $$i = \omega_0 q_m \sin\omega_0 t$$

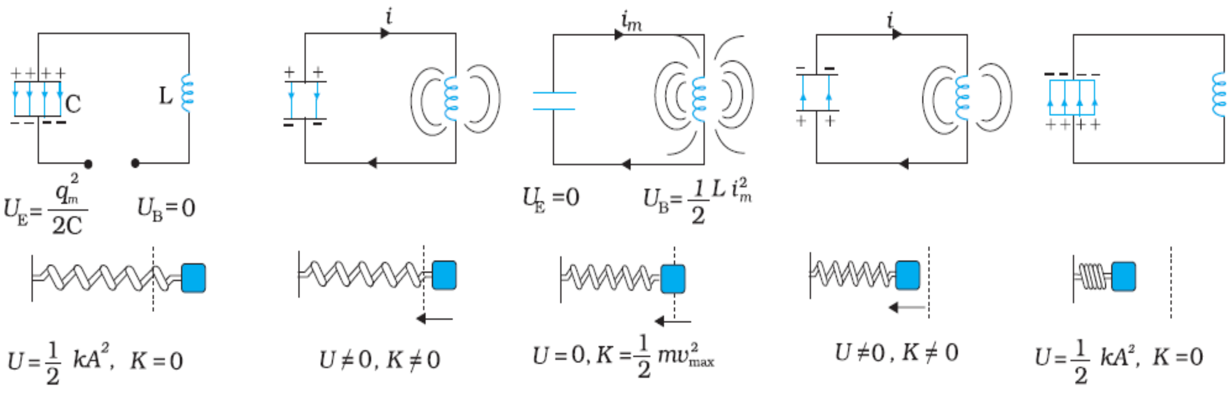
(or) $$\boxed{i = i_m \sin\omega_0 t} \longrightarrow (7)$$

where, $i_m = \omega_0 q_m$

Diagrammatic Representation of LC oscillations:

A capacitor of capacitance C with initial charge q_m is connected to an inductor as shown below.

The oscillations in an LC circuit are analogous to the oscillations of a block at the end of a spring.



(a) At the initial stage, when the switch is opened, the energy stored in the capacitor is maximum. It is stored in the form of '**electrostatic potential energy** (U_E).

It is given by $U_E = \frac{1}{2}\frac{q_m^2}{C}$.

As there is no current in the inductor, the energy stored in the inductor (U_B) is equal to zero. Thus, the total energy of the LC circuit is

$$U = U_E + U_B = \frac{1}{2}\frac{q_m^2}{C} + 0 = \frac{1}{2}\frac{q_m^2}{C}$$

(b) At a time t = 0, the switch is closed and the capacitor starts to discharge.

As the current increases, it sets up a magnetic field in the inductor and thereby, some energy gets stored in the inductor in the form of **magnetic energy**. It is given by

$$U_B = \frac{1}{2} L i^2$$

(c) As the current reaches its maximum value i_m (at $t = T/4$) all the energy is stored in the magnetic field. It is given by $U_B = \frac{1}{2} L i_m^2$

Now, the capacitor has no charge and hence no energy. ($U_E = 0$)

(d) The magnetic field collapses and the current now starts charging the capacitor so that some energy will be stored in the capacitor and the remaining energy is stored in the inductor. ($U_E \neq 0$ and $U_B \neq 0$)

(e) The process continues till the capacitor is fully charged (at $t = T/2$) . But it is charged with a polarity opposite to its initial state in Fig.(a).

The whole process just described will now repeat itself till the system reverts to its original state. Thus, the energy in the system oscillates between the capacitor and the inductor.

NOTE: **The above discussion of LC oscillations is not realistic for two reasons**:

(i) Every inductor has some resistance. The effect of this resistance is to introduce a damping effect on the charge and current in the circuit and the oscillations finally die out.

(ii) Even if the resistance were zero, the total energy of the system would not remain constant. It is radiated away from the system in the form of electromagnetic waves. In fact, radio and TV transmitters depend on this radiation.

TRANSFORMER:

Transformer is a device used **to transform electrical power** from one circuit to another without changing its frequency. The applied alternating **voltage is either increased or decreased** with corresponding **decrease or increase of current** in the circuit.

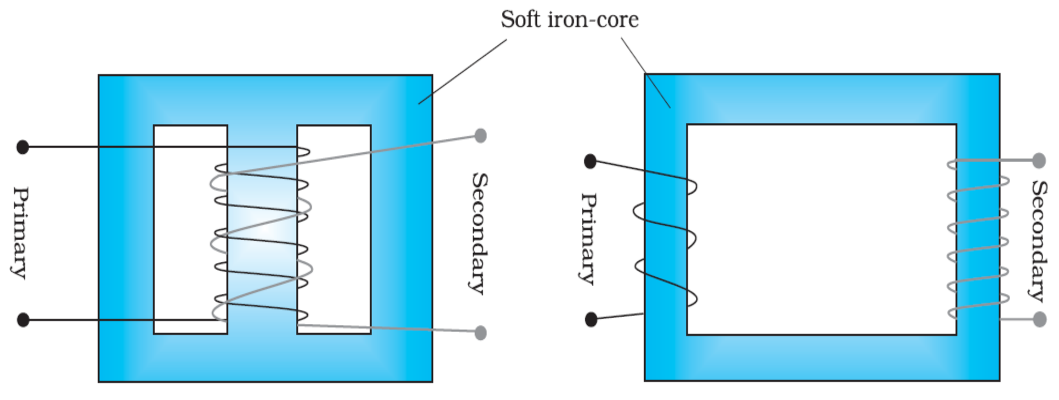
Principle:

The principle of transformer is the **mutual induction** between two coils.

Construction:

A transformer consists of two sets of coils, insulated from each other. They are wound on a soft-iron core, either one on top of the other as in Fig. (a) or on separate limbs of the core as in Fig. (b).

One of the coils called the ***primary coil*** has Np turns. The other coil is called the ***secondary coil***; it has Ns turns. Often the primary coil is the input coil and the secondary coil is the output coil of the transformer.



(a) (b)

When an alternating voltage is applied to the primary, the resulting current produces an alternating magnetic flux which links the secondary and induces an emf in it. The value of this emf depends on the number of turns in the secondary. We consider an ideal transformer in which the primary has negligible resistance and all the flux in the core links both primary and secondary windings. Let ϕ be the flux in each turn in the core at time *t* due to current in the primary when a voltage v_p is applied to it.

Then the induced emf or voltage ε_s in the secondary coil with N_s turns is given by

$$\varepsilon_s = -N_s \frac{d\phi}{dt} \longrightarrow (1)$$

The alternating flux φ also induces an emf, called back emf in the primary. This is given by

$$\varepsilon_p = -\,N_p \frac{d\phi}{dt} \longrightarrow (2)$$

Since the primary resistance is not zero and primary current is not infinite, $\varepsilon_p = v_p$ and $\varepsilon_s = v_s$.

$$\therefore \quad v_s = -\,N_s \frac{d\phi}{dt} \longrightarrow (3)$$

$$\text{and} \quad v_p = -\,N_p \frac{d\phi}{dt} \longrightarrow (4)$$

Dividing equn (3) by equn (4),

$$\frac{v_s}{v_p} = \frac{N_s}{N_p} \longrightarrow (5)$$

The above relation has been obtained using three assumptions:

(i) the primary resistance and current are small.

(ii) the same flux links both the primary and the secondary as very little flux escapes from the core,

(iii) the secondary current is small.

For an ideal transformer, since there is no power loss,

Input power = Output power

$$i_p\, v_p = i_s\, v_s$$

$$\text{(or)} \quad \frac{i_p}{i_s} = \frac{v_s}{v_p} \longrightarrow (6)$$

Combining equns (5) and (6), we get

$$\boxed{\frac{i_p}{i_s} = \frac{v_s}{v_p} = \frac{N_s}{N_p} = K} \longrightarrow (7)$$

where, K → constant known as '**voltage transformation ratio'**.

Case (i): If **K > 1**, $N_s > N_p$ and hence $v_s > v_p$ and $i_s < i_p$.

This is the case of **step-up transformer** in which voltage is increased and the corresponding current is decreased.

Case (ii): If **K < 1**, $N_s < N_p$ and hence $v_s < v_p$ and $i_s > i_p$.

This is the case of **step-down transformer** in which voltage is decreased and the corresponding current is increased.

Efficiency of a transformer: (η)

The efficiency η of a transformer is defined as the ratio of the useful output power to the input power.

$$\boxed{\eta = \frac{\text{output power}}{\text{input power}} \times 100\ \%}$$

<u>Note</u>: Actual Transformers have efficiency ranging from 96% to 99% but never equal to 100%, due to various types of energy losses taking place in it.

<u>ENERGY LOSSES IN A TRANSFORMER</u>:

1. **<u>Flux Leakage</u>**:
There is always some flux leakage; that is, not all of the flux due to primary passes through the secondary due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.
2. **<u>Copper loss:</u>**
Transformer windings have electrical resistance. When an electric current flows through them, some amount of energy is dissipated due to **Joule heating** I^2R. This energy loss is called copper loss. In high current, low voltage windings, these are minimised by using thick wire.
3. **<u>Eddy current loss</u>**: The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by having a laminated core.(Ex: Stelloy)
4. **<u>Hysteresis loss</u>**: The magnetisation of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss. (Ex: Mumetal and Silicon Steel)

BEST WISHES

<u>BIBLIOGRAPHY</u>:

I acknowledge that the contents are taken from NCERT Text book in Physics for class XII (2020-21)

<u>PREPARED BY</u>:

P.SARAVANAN
PGT (SS)
AECS, MYSORE