ATOMIC ENERGY CENTRAL SCHOOL, MYSORE

DISTANT LEARNING PROGRAM 2020-21

CHAPTER-7

ALTERNATING CURRENT

PPT- (1 of 2)

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INTRODUCTION:

An alternating voltage is the voltage which changes polarity at regular intervals of time and the direction of the resulting alternating current also changes accordingly.

If the wave form of alternating voltage is a sine wave, then it is known as " **sinusoidal alternating voltage** ". It is given by

$$v = v_m \sin \omega t$$

where, $v \rightarrow$ instantaneous value of alternating voltage

$v_m \rightarrow$ voltage amplitude (or) maximum value of alternating voltage

$\omega \rightarrow$ angular frequency of alternating voltage

When sinusoidal alternating voltage is applied to a closed circuit, the resulting current also is sinusoidal in nature.

It is given by the relation

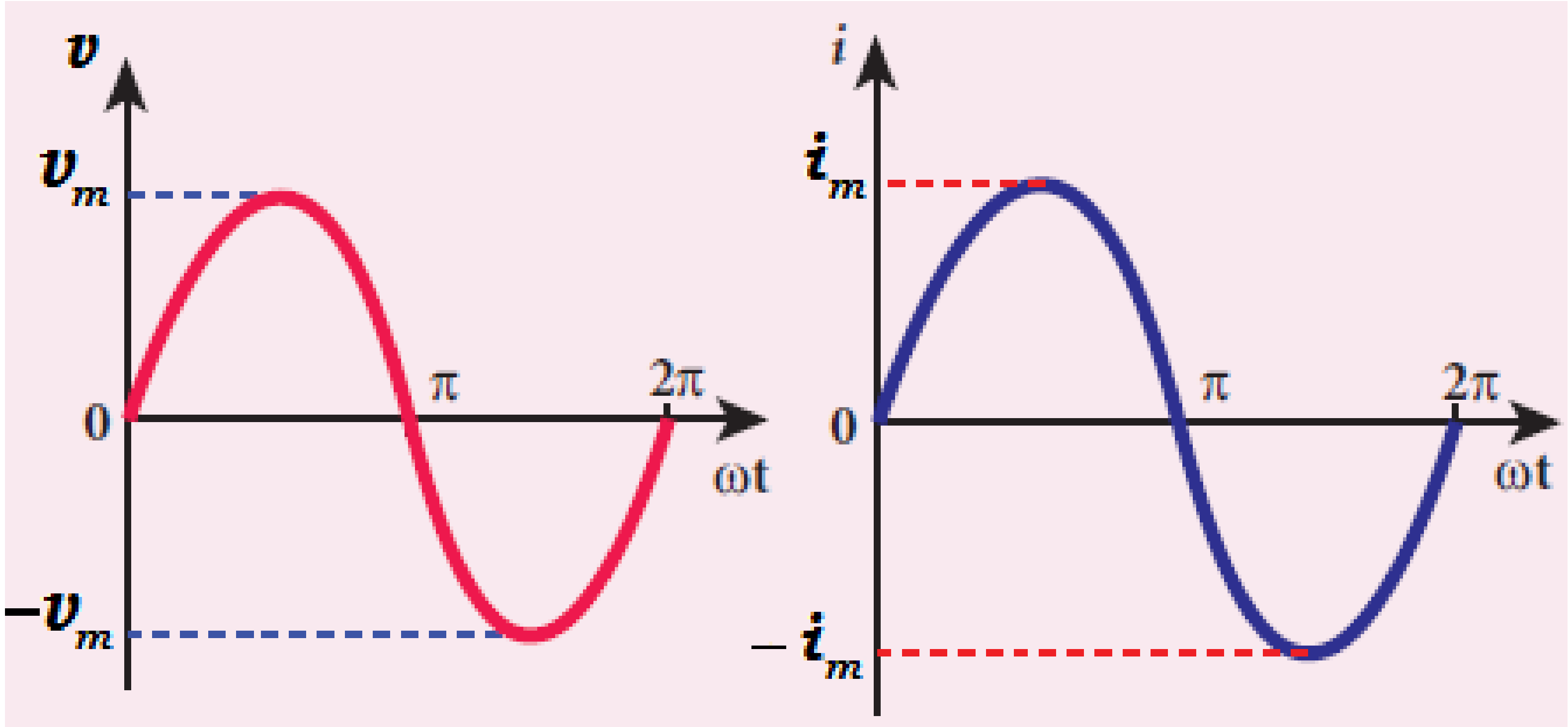
$$i = i_m \sin \omega t$$

where, $i \rightarrow$ instantaneous value of alternating current

$i_m \rightarrow$ current amplitude (or) maximum value of alternating current

$\omega \rightarrow$ angular frequency of alternating current

The direction of sinusoidal voltage or current is reversed after every half-cycle and its magnitude is also changing continuously as shown in figure below.



Alternating voltage **Alternating current**

MEAN (OR) AVERAGE VALUE OF AC: (i_{av})

Alternating current in a circuit changes from one instant to another instant and its direction also reverses for every half cycle.

During positive half cycle, current is taken as positive and during negative half cycle, current is taken as negative. Therefore, the mean value or **average value of alternating current over one complete cycle is zero.**

Therefore, the average or **mean value of AC is measured over one half of cycle.**

DEFINITION:

Average value of alternating current is defined as the average of all values of current over a positive half cycle or negative half cycle.

It can be shown that $i_{av} = 0.637\, i_m$ for positive half cycle and

$i_{av} = -0.637\, i_m$ for negative half cycle

RMS value of AC: (I_{rms})

Root Mean Square value of AC is defined as the square root of the mean value of the squares of all currents over one cycle.

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n}}$$

It is also defined as that value of the steady current (DC), which when passed through a resistor for a given time, will generate the same amount of heat as generated by an alternating current when passed through the same resistor for the same time.

It is also called as **" effective value of AC "** (I_{eff}) or (I)

It can be shown that, $$I_{rms} = \frac{i_m}{\sqrt{2}} = 0.707\ i_m$$

ie, rms value of AC = 70.7 % of current amplitude

Similarly, $$V_{rms} = \frac{v_m}{\sqrt{2}} = 0.707\ v_m$$

AC VOLTAGE APPLIED TO A RESISTOR:

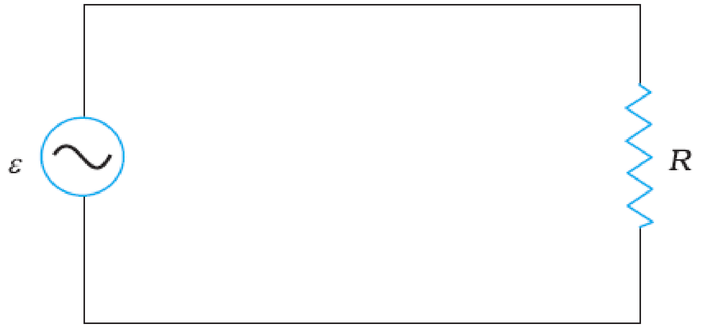
Let an alternating source of potential difference (ε) be connected across a resistor of resistance R.

The instantaneous value of the applied potential difference is given by

$$v = v_m \sin \omega t \longrightarrow (1)$$

where, $v_m \rightarrow$ voltage amplitude of the oscillating potential difference

$\omega \rightarrow$ angular frequency of oscillating potential difference



An alternating current i flowing in the circuit due to this voltage, develops a potential drop across R and is given by

$$v = i R \longrightarrow (2)$$

According to Kirchhoff's (loop rule)voltage law, the algebraic sum of potential differences in a closed circuit is zero.

$$\sum \varepsilon = 0$$

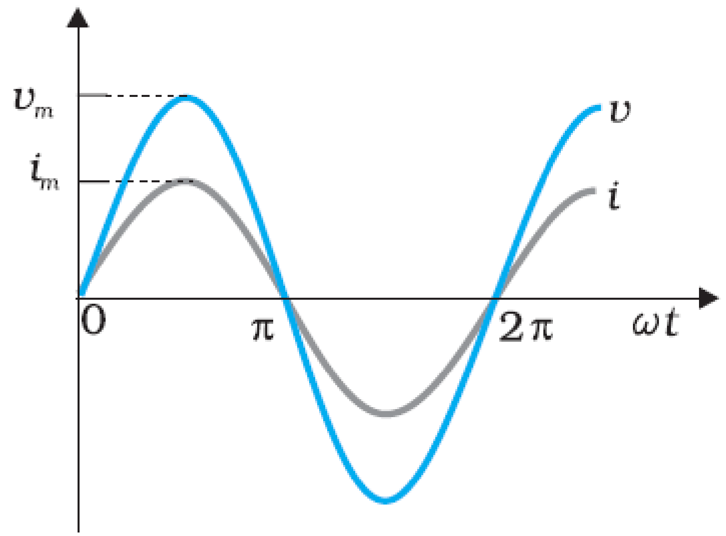
$$i\,\mathrm{R} = v_m \sin \omega t$$

(or) $$i = \frac{v_m}{\mathrm{R}} \sin \omega t$$

(or) $$\boldsymbol{i = i_m \sin \omega t} \longrightarrow (3)$$

where, $i_m = \frac{v_m}{\mathrm{R}}$ = current amplitude (From Ohm's law)

From equations (1) and (3) , it is found that the current and voltage reach their maximum value, minimum value and zero value simultaneously. Hence, current and voltage are in phase with each other.



The sum of the instantaneous current values over one complete cycle is zero, and the average current over one cycle of AC is zero. But the sum of the squares of all currents over one cycle is not zero.

Since i^2 is always positive, irrespective of whether *i* is positive or negative, Joule heating and dissipation of electrical energy takes place when an ac current passes through a resistor.

The instantaneous power dissipated in the resistor is given by

$$p = i^2 \mathrm{R} = (i_m \sin \omega t)^2 \mathrm{R}$$

$$= {i_m}^2 \mathrm{R} \sin^2 \omega t$$

The average power over a cycle of AC is

$$\bar{p} = \langle i^2 R \rangle = \langle {i_m}^2 \mathrm{R}\, sin^2\, \omega t \rangle \longrightarrow \quad (4)$$

Since ${i_m}^2$ and R are constants,

$$\bar{p} = {i_m}^2 \mathrm{R} \langle \sin^2 \omega t \rangle$$

But, $\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$

$$= \frac{1}{2} - \frac{\cos 2\omega t}{2}$$

Since $\langle \cos 2\omega t \rangle = 0$

Hence, $\langle \sin^2 \omega t \rangle = \frac{1}{2}$

$$\therefore \quad \bar{p} = \frac{i_m{}^2 R}{2} \longrightarrow (5)$$

The same amount of DC power ($P = I^2 R$) will be dissipated ,if a current equal to I_{rms} or I flows through the resistor R.

$$\therefore \quad \mathbf{P = I_{rms}{}^2 R} \longrightarrow (6)$$

But, $\quad P = \bar{p}$

$$I_{rms}^{\ 2}\,R = \frac{i_m^{\ 2}\,R}{2}$$

$$I_{rms}^{\ 2} = \frac{i_m^{\ 2}}{2}$$

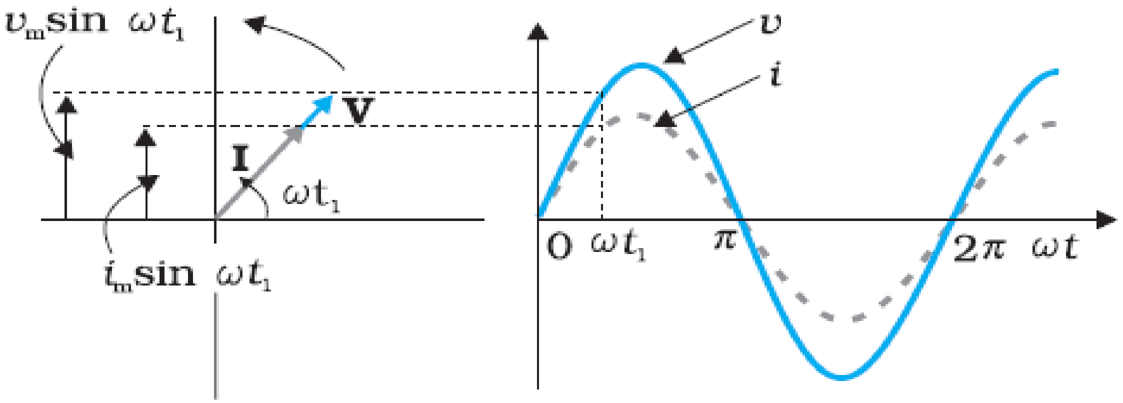
(or) $$\mathbf{I = I_{rms} = \frac{i_m}{\sqrt{2}} = 0.707\ i_m} \longrightarrow (7)$$

Similarly, RMS value of voltage or effective value of voltage is given by

$$\mathbf{V = V_{rms} = \frac{v_m}{\sqrt{2}} = 0.707\ v_m} \longrightarrow (8)$$

REPRESENTATION OF AC CURRENT AND VOLTAGE:

PHASOR: A phasor* is a vector which rotates in the anti clockwise direction about the origin with angular speed ω, as shown in figure. The vertical components of phasors **V** and **I** represent the sinusoidally varying quantities *v* and *i*.



1. The length of the line segment equals the peak value v_m (or i_m) of the alternating voltage (or current).

2. Its angular velocity ω is equal to the angular frequency of the alternating voltage (or current)

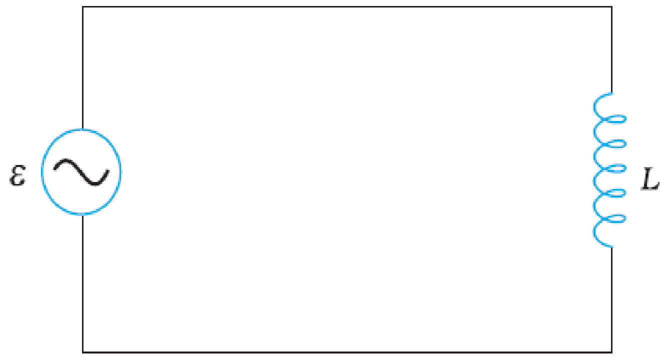
3.The projection of phasor on any vertical axis gives the instantaneous value of the alternating voltage (or current)

4.The angle between the phasor and the axis of reference (positive x-axis) indicates the phase of the alternating voltage (or current).

PHASOR DIAGRAM:

The diagram which represents the current and voltage phasors, and the phase relationship between voltage and current is called a Phasor diagram.

AC VOLTAGE APPLIED TO AN INDUCTOR:



Consider a circuit containing a pure inductor of inductance L connected across an alternating voltage source ε as shown in the figure. The alternating voltage is given by the equation

$$v = v_m \sin \omega t \longrightarrow (1)$$

The alternating current flowing through the inductor induces a self-induced emf or back emf in the circuit which is given by

$$v' = -L\frac{di}{dt} \longrightarrow (2)$$

According to Kirchhoff’s (loop rule)voltage law, the algebraic sum of potential differences in a closed circuit is zero

ie, , $\sum \varepsilon = 0$

$$\therefore \qquad v - v' = 0$$

(or) $$v_m \sin \omega t - L\frac{di}{dt} = 0$$

(or) $$L\frac{di}{dt} = v_m \sin \omega t$$

(or) $$\boldsymbol{\frac{di}{dt} = \frac{v_m}{L} \sin \omega t} \longrightarrow \quad (3)$$

Since current i is a function of time, the slope $\frac{di}{dt}$ must be a sinusoidally varying quantity.Hence, the current in the circuit is got by integrating equn (3) with respect to time.

$$\int \frac{di}{dt}\,dt = \frac{v_m}{L} \int \sin \omega t \; dt$$

(or) $$i = \frac{v_m}{L} \frac{(-\cos \omega t)}{\omega} + \text{constant.}$$

Since the current oscillates symmetrically about zero, the time independent component of current is zero. Hence, the integration constant is zero.

(or) $$i = \frac{v_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \left(\because -\cos \omega t = \sin\left(\omega t - \frac{\pi}{2}\right) \right)$$

(or) $$\boldsymbol{i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)} \longrightarrow (4)$$

where, $\frac{v_m}{\omega L} = i_m$ = current amplitude (maximum value of current)

The term ω L is analogous to resistance and is called as "inductive reactance" (X_L). It is the opposition offered by the inductor to the flow of AC.

$$\therefore \qquad X_L = \omega L \quad \text{(Its unit is ohm)}$$

(or) $$X_L = 2\pi \upsilon L$$

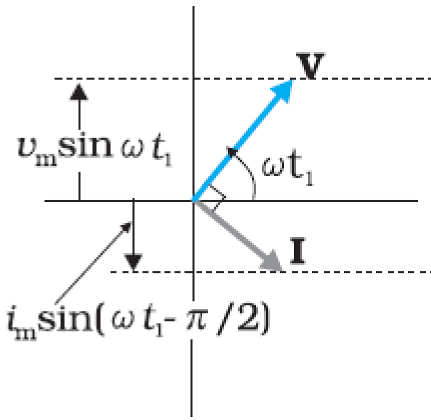
where, υ → frequency of the applied AC voltage.

Hence, the amplitude of the current is given by $i_m = \frac{v_m}{X_L}$

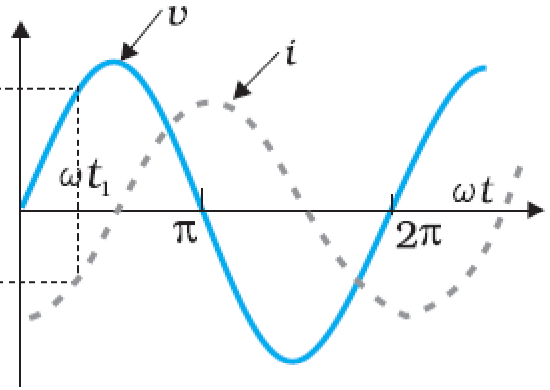
For AC, $X_L \propto \upsilon$ (X_L is directly proportional to the frequency)

For DC, $\upsilon = 0$. Hence, $X_L = 0$. Hence, **an ideal inductor offers no resistance to DC** current.

From equns (1) and (4), it is found that, **the current lags the voltage by a phase angle $\frac{\pi}{2}$** or one quarter of a cycle.



Phasor diagram



voltage versus current graph

We see that the current reaches its maximum value **later** than voltage by one fourth of a period $\frac{T}{4}$.

Also, $\frac{T}{4} = \frac{\pi/2}{\omega}$

Instantaneous Power: (p_L)

The instantaneous power supplied to the inductor is equal to the product of the instantaneous current and instantaneous voltage.

$$p_L = i\,v = i_m \sin\left(\omega t - \frac{\pi}{2}\right) \times v_m \sin \omega t$$

$$= i_m v_m\,(-\cos \omega t) \sin \omega t$$

$$= -\,i_m v_m\,(\cos \omega t)(\sin \omega t)$$

$$= \frac{-\,i_m v_m}{2}\,(2 \sin \omega t \cos \omega t)$$

$$= \frac{-\,i_m v_m}{2}\,(\sin 2\omega t)$$

So, the average power over one cycle of AC is

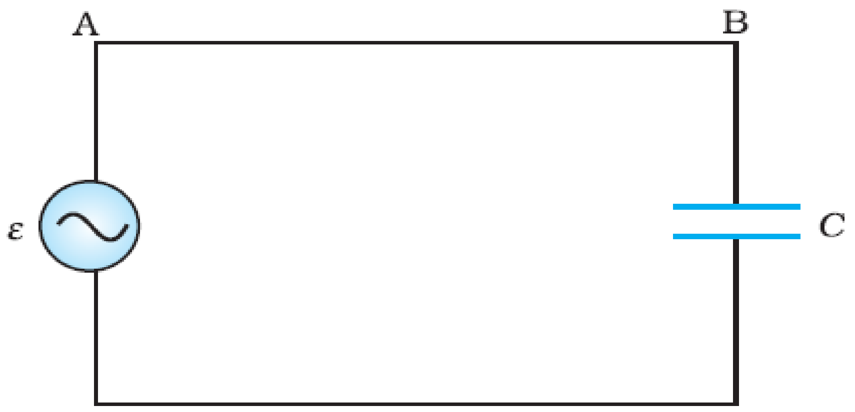
$$P_L = \left\langle \frac{-\,i_m v_m}{2}\,(\sin 2\omega t) \right\rangle$$

$$= \frac{-\,i_m v_m}{2} \langle \sin 2\omega t \rangle$$

Since $\langle \sin 2\omega t \rangle = 0$, $P_L = 0$

Thus, the average power supplied to an inductor over one complete cycle of AC is zero.

AC VOLTAGE APPLIED TO A CAPACITOR:



Consider a circuit containing a capacitor of capacitance C connected across an alternating voltage source.

The alternating voltage is given by

$$v = v_m \sin \omega t \longrightarrow (1)$$

The capacitor is alternately charged and discharged as the current reverses each half cycle. Let *q* be the charge on the capacitor at any time *t*. The instantaneous voltage *v* across the capacitor is

$$v = \frac{q}{C} \longrightarrow (2)$$

According to Kirchhoff's (loop rule)voltage law, the voltage across the source and the capacitor are equal,

$$\therefore \quad \frac{q}{C} = v_m \sin \omega t$$

$$\text{(or)} \quad q = v_m C \sin \omega t \longrightarrow (3)$$

The instantaneous current in the circuit is given by

$$i = \frac{dq}{dt} = \frac{d}{dt}(v_m \, \mathrm{C} \; \sin \omega t)$$

(or) $$i = v_m \, \mathrm{C} \, \frac{d}{dt}(\sin \omega t)$$

$$= v_m \, \mathrm{C} \; (\cos \omega t)\, \omega$$

(or) $$i = \frac{v_m}{1/\omega \mathrm{C}} \cos \omega t$$

(or) $$i = \frac{v_m}{1/\omega \mathrm{C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

(or) $$\boldsymbol{i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)} \longrightarrow (4)$$

where, $\frac{v_m}{1/\omega \mathrm{C}} = i_m$ = current amplitude (maximum value of current)

The term $(1/\omega \mathrm{C})$ is analogous to resistance and is called as "capacitive reactance" (X_C). It is the opposition offered by the capacitor to the flow of AC.

∴ $$X_\mathrm{C} = \frac{1}{\omega \mathrm{C}} \qquad \text{(Its unit is ohm)}$$

(or) $$X_\mathrm{C} = \frac{1}{2\,\pi\,\upsilon\,\mathrm{C}}$$

where, $\upsilon \rightarrow$ frequency of the applied AC voltage.

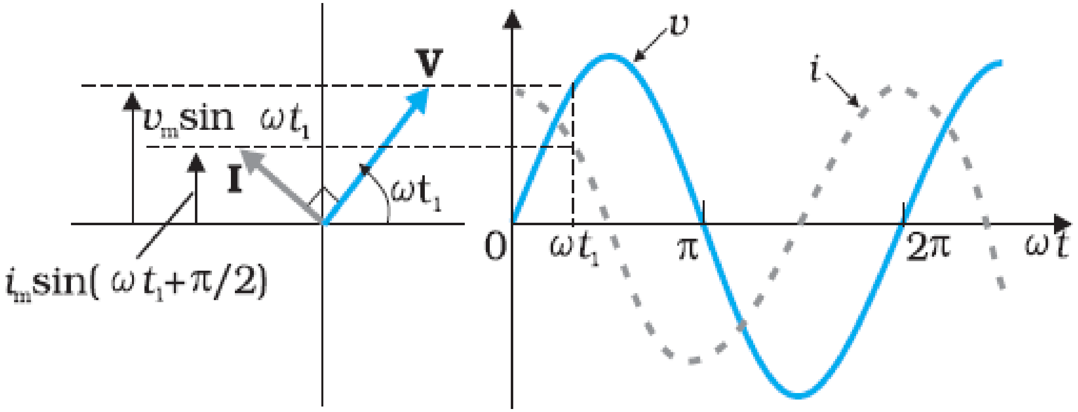
Hence, the amplitude of the current is given by $i_m = \frac{v_m}{X_\mathrm{C}}$

For AC, $X_C \propto \frac{1}{\upsilon}$ (X_C is inversely proportional to the frequency)

For DC, $\upsilon = 0$. Hence, $X_C = \frac{1}{0}$ = infinity.

Thus a **capacitive circuit offers infinite resistance to the steady current (DC)**. So the steady current (DC) cannot flow through the capacitor.

From equns (1) and (4), it is found that, **the current leads the voltage by a phase angle $\frac{\pi}{2}$** or one quarter of a cycle.



Phasor diagram **voltage versus current graph**

We see that the current reaches its maximum value **earlier** than voltage by one fourth of a period $\frac{T}{4}$.

Also, $\frac{T}{4} = \frac{\pi/2}{\omega}$

Instantaneous Power: (p_C)

The instantaneous power supplied to the capacitor is equal to the product of the instantaneous current and instantaneous voltage.

$$p_c = i\,v = i_m \sin\left(\omega t + \frac{\pi}{2}\right) \times v_m \sin \omega t$$

$$= i_m\, v_m\, (\cos \omega t)\, (\sin \omega t)$$

$$= \frac{i_m\, v_m}{2}\, (2 \sin \omega t \cos \omega t)$$

$$= \frac{i_m\, v_m}{2}\, (\sin 2\, \omega t)$$

So, the average power over one cycle of AC is

$$P_C = \left\langle \frac{i_m\, v_m}{2}\, (\sin 2\, \omega t) \right\rangle$$

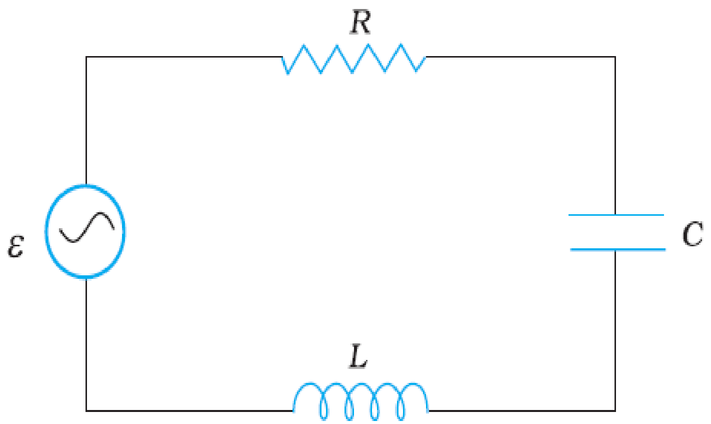
$$= \frac{i_m\, v_m}{2} \langle \sin 2\, \omega t \rangle$$

Since $\langle \sin 2\, \omega t \rangle = 0$, $P_C = 0$

Thus, the average power supplied to a capacitor over one complete cycle of AC is zero.

AC VOLTAGE APPLIED TO A SERIES LCR CIRCUIT:

Consider a circuit containing a resistor of resistance R, an inductor of inductance L and a capacitor of capacitance C connected across an alternating voltage source ε.



The applied alternating voltage of the source is given by the equation.

$$v = v_m \sin \omega t \longrightarrow (1)$$

Also, $v = v_L + v_C + v_R$

$$= L\frac{di}{dt} + \frac{q}{C} + iR \longrightarrow (2)$$

To determine the instantaneous current and its phase relationship with applied alternating voltage v,we use the phasor diagram method.

Phasor Diagram Method:

Since the inductor, capacitor and resistor are connected in series, the current flowing through each element will be the same at any instant. It is given by

$$i = i_m \sin(\omega t + \phi) \longrightarrow \quad (3)$$

where, $\phi \rightarrow$ phase difference between the voltage across the source and the current in the circuit.

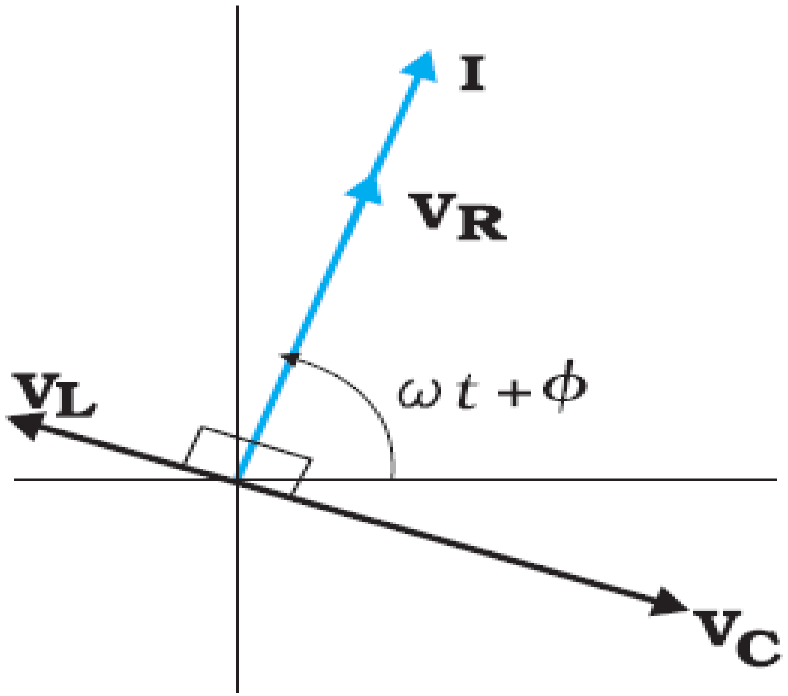
Let **I** be the phasor representing the current in the circuit.

Let $\mathbf{V_R} \rightarrow$ voltage across the resistor. ($\mathbf{V_R}$ is in phase with **I**)

Let $\mathbf{V_L} \rightarrow$ voltage across the inductor. ($\mathbf{V_L}$ is ahead of **I** by $\frac{\pi}{2}$)

Let $\mathbf{V_C} \rightarrow$ voltage across the capacitor. ($\mathbf{V_C}$ is behind **I** by $\frac{\pi}{2}$)

Let $\mathbf{V} \rightarrow$ voltage across the source.



Amplitude(length) of phasor $\mathbf{V_R} = v_{Rm} = i_m R$

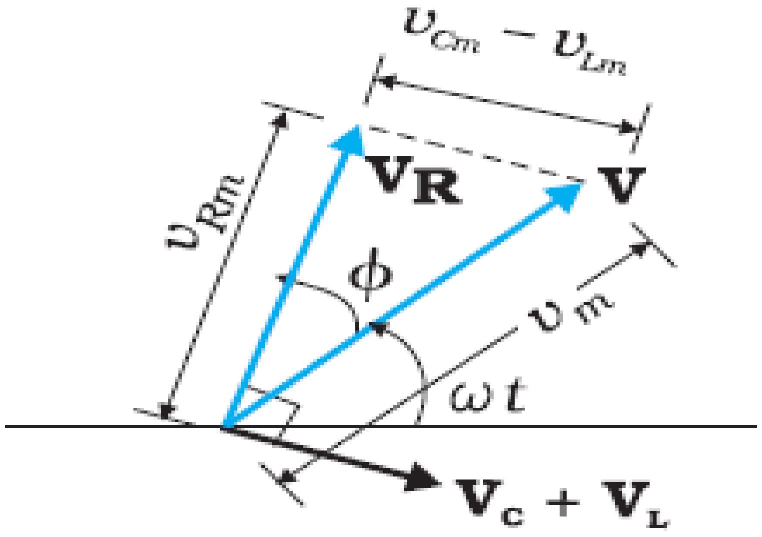
Amplitude(length) of phasor $\mathbf{V_L} = v_{Lm} = i_m X_L$

Amplitude(length) of phasor $\mathbf{V_C} = v_{Cm} = i_m X_C$

The voltage equation for the LCR circuit is

$$v = v_L + v_C + v_R \longrightarrow (4)$$

The phasor relation is $$\mathbf{V} = \mathbf{V_L} + \mathbf{V_R} + \mathbf{V_C} \longrightarrow (5)$$



Let the circuit be effectively capacitive, ie, $V_C > V_L$

V_C and V_L are always along the same line and in opposite directions. They can be combined into a single phasor $V_C + V_L$ which has a magnitude $|v_{C\,m} - v_{L\,m}|$.

Since **V** is represented as the hypotenuse of a right-triangle whose sides are V_R and $(V_C + V_L)$, then from Pythagorean theorem,

$$v_m^{\,2} = v_{R\,m}^{\,2} + (v_{C\,m} - v_{L\,m})^2 \longrightarrow \quad (6)$$

Substituting $v_{R\,m} = i_m R$, $v_{C\,m} = i_m X_C$ and $v_{L\,m} = i_m X_L$, we get

$$v_m^{\,2} = (i_m R)^2 + (i_m X_C - i_m X_L)^2$$

$$= i_m^{\,2}\,[R^2 + (X_C - X_L)^2]$$

(or)

$$v_m = i_m \sqrt{R^2 + (X_C - X_L)^2} \longrightarrow \quad (7)$$

$$\therefore \quad i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}} \longrightarrow \quad (8)$$

Comparing equn (8) with Ohm's law, it is found that the factor $\sqrt{R^2 + (X_C - X_L)^2}$ plays the role of resistance. This **effective opposition** offered by the series LCR circuit to the flow of AC is called **impedance** (Z)

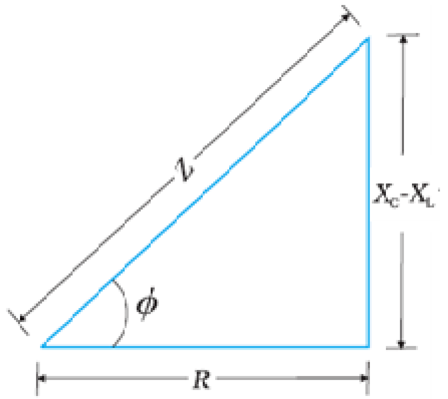
$$\therefore \quad i_m = \frac{v_m}{Z} \longrightarrow \quad (9)$$

and

$$Z = \sqrt{R^2 + (X_C - X_L)^2} \longrightarrow \quad (10)$$

Since phasor **I** is always parallel to phasor $\mathbf{V_R}$, the phase angle ϕ is the angle between $\mathbf{V_R}$ and **V** .

It can be determined from the following **impedance diagram**.



$$\tan \phi = \frac{(X_C - X_L)}{R} \longrightarrow \qquad (11)$$

Also,

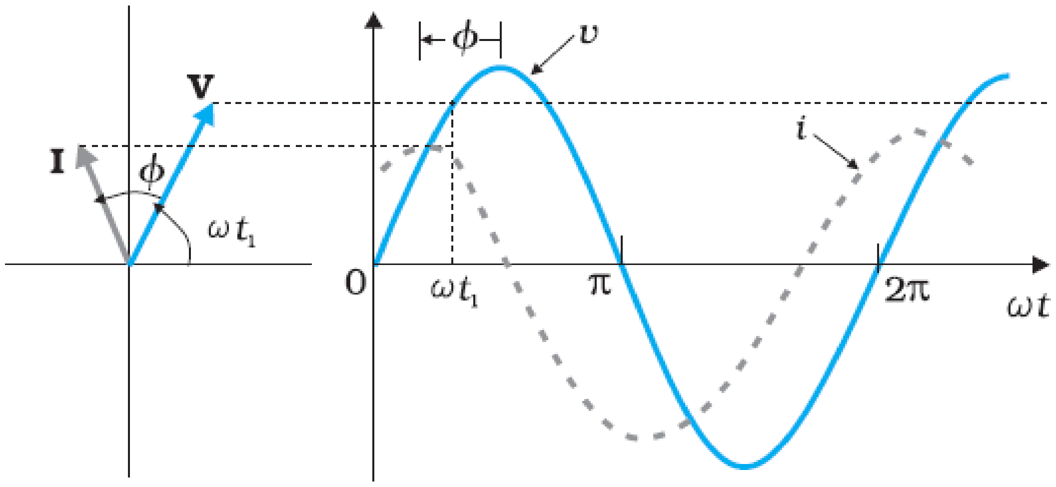
$$\tan \phi = \frac{(v_{C\,m} - v_{L\,m})}{v_{Rm}} \longrightarrow \qquad (12)$$

If $X_C > X_L$, then ϕ is positive and the circuit is predominantly capacitive.

If $X_C < X_L$, then ϕ is negative and the circuit is predominantly inductive.

If $X_C = X_L$, then $\phi = 0$. Voltage and Current are in phase with each other.

The phasor diagram and variation of v and i with ωt for an arbitrary time t_1 , for the case $X_C > X_L$ is shown below .



The solution so obtained by drawing phasor diagram is called **‘steady state solution’**.

END OF PPT-1
IN
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BIBLIOGRAPHY:

I acknowledge that the contents are taken from NCERT Text book in Physics for class XII (2020-21)

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