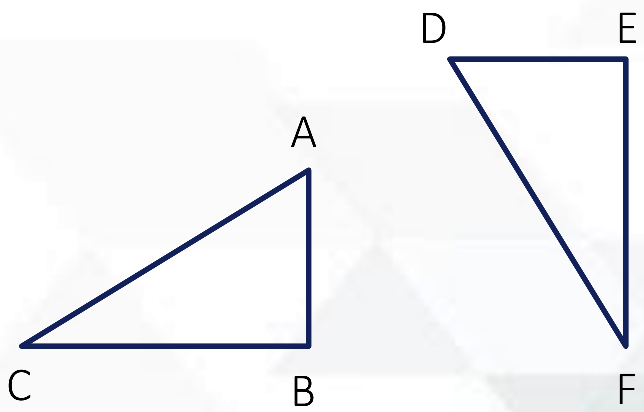
TRIANGLES

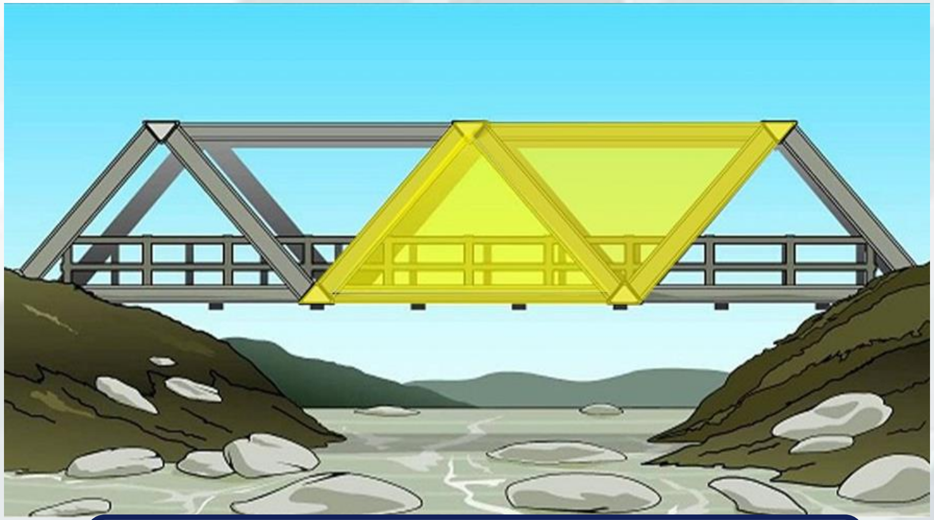
CONTENTS

CONGRUENCE

Two geometric figures with exactly same shape and size



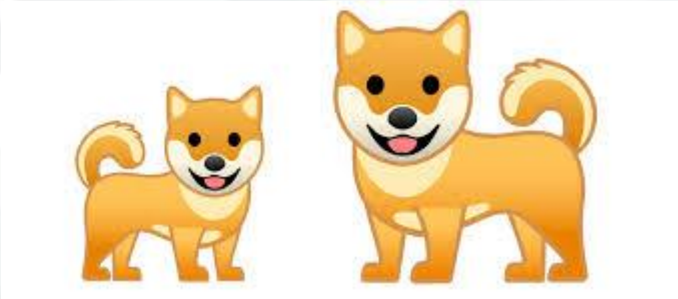
$\Delta ABC \cong \Delta DEF$

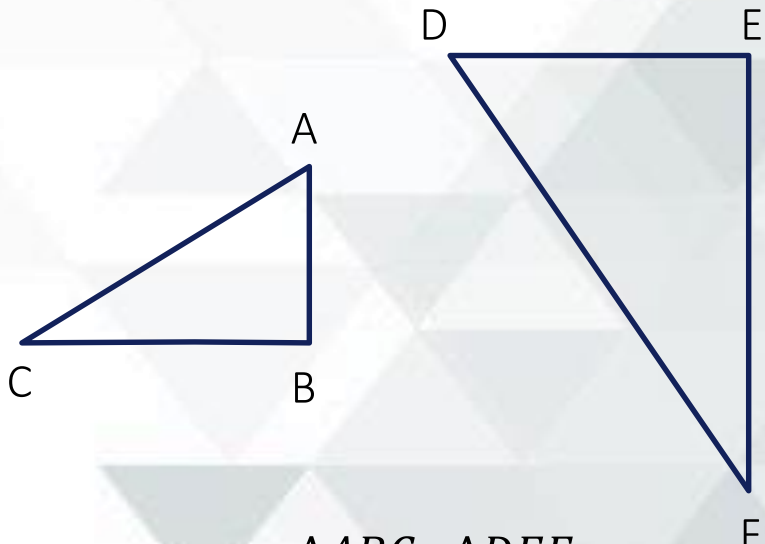


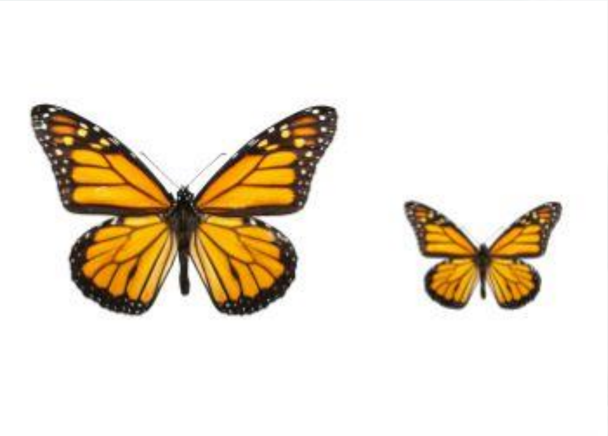
Congruent Triangles in Bridges

SIMILAR FIGURES

Two figures having same shape and not necessarily the same size are called ***similar figures***



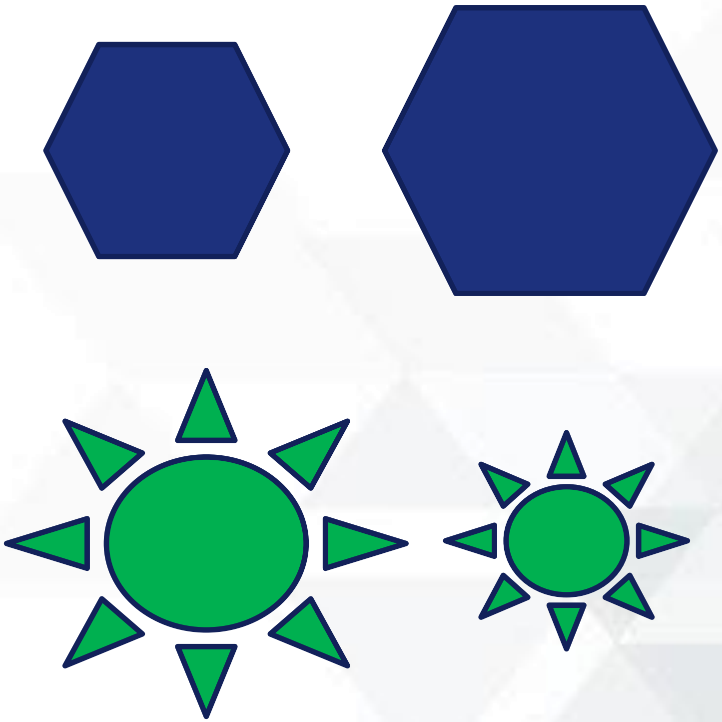




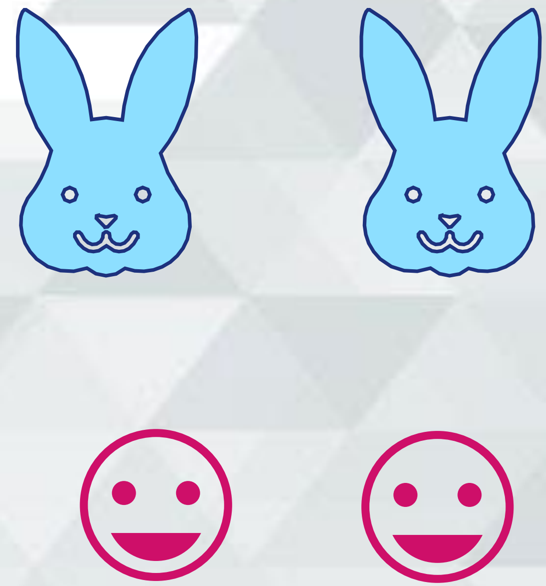
$\Delta ABC \sim \Delta DEF$

SIMILAR vs CONGRUENCE

Similarity

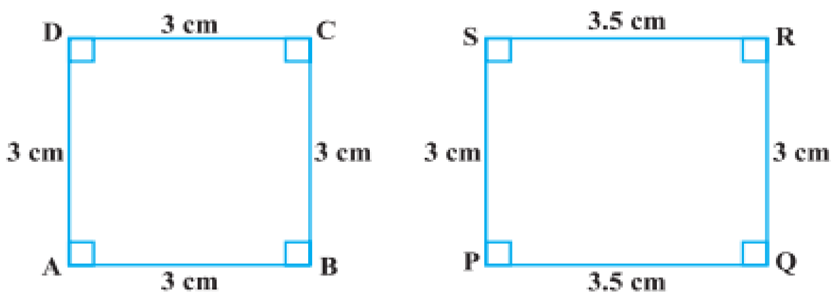


Congruence

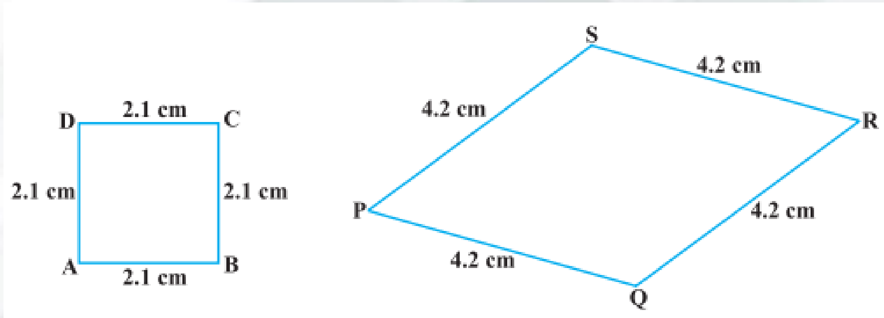


All congruent figures are similar but all similar figures need not be congruent

SIMILARITY OF POLYGONS



In the two quadrilaterals (a square and a rectangle), corresponding angles are equal, but their corresponding sides are not in the same ratio. So, the two quadrilaterals are not similar

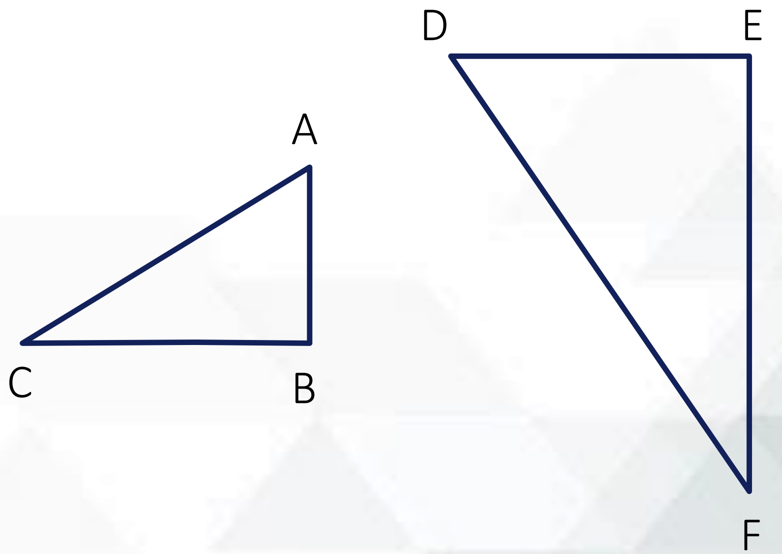


In the two quadrilaterals (a square and a rhombus), corresponding sides are in the same ratio, but their corresponding angles are not equal. Again, the two polygons (quadrilaterals) are not similar.

SIMILARITY OF POLYGONS

- Two polygons of the same number of sides are similar, if
  - their corresponding angles are equal **and**
  - their corresponding sides are in the same ratio (or proportion)

SIMILARITY OF TRIANGLES



$\Delta ABC \sim \Delta DEF$

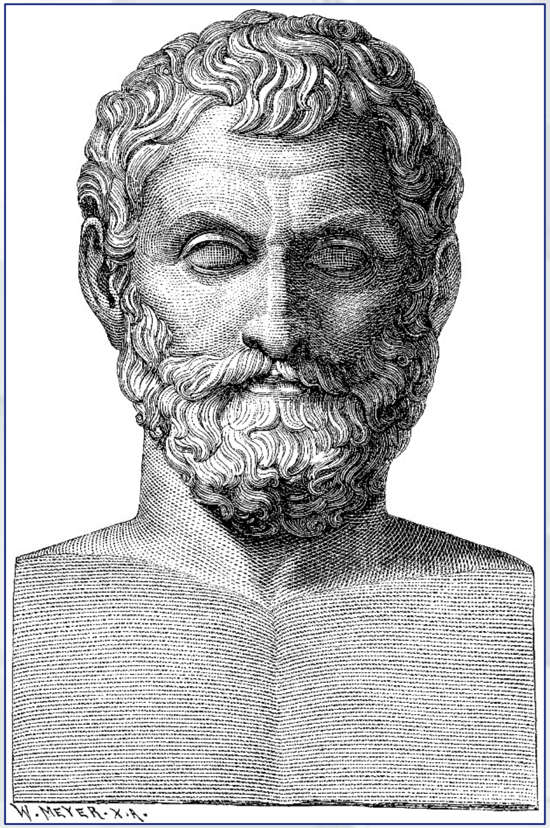
$\Delta\mathrm{ABC}$ and $\Delta\mathrm{DEF}$ are similar to each other.

- ❖ ∠A = ∠D
- ❖ ∠B = ∠E
- ❖ ∠C = ∠F
- ❖ $\frac{\mathbf{AB}}{\mathbf{DE}} = \frac{\mathbf{BC}}{\mathbf{EF}} = \frac{\mathbf{AC}}{\mathbf{DF}}$

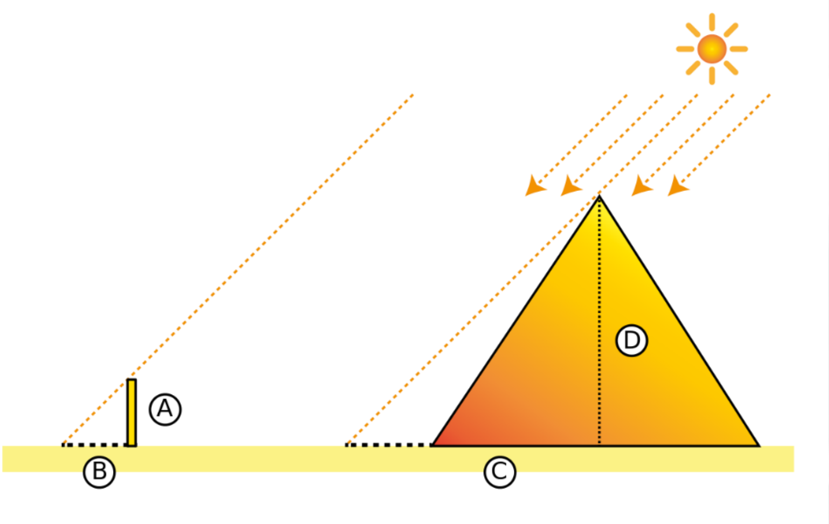
It must be noted that as done in the case of congruency of two triangles, the similarity of two triangles should also be expressed symbolically, using correct correspondence of their vertices. For example, for the triangles ABC and DEF, we cannot write Δ ABC ~ Δ EDF or Δ ABC ~ Δ FED. However, we can write Δ BAC ~ Δ EDF

BASIC PROPORTIONALITY THEOREM (or) THALES THEOREM

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio



DISCOVERY OF THALES THEOREM

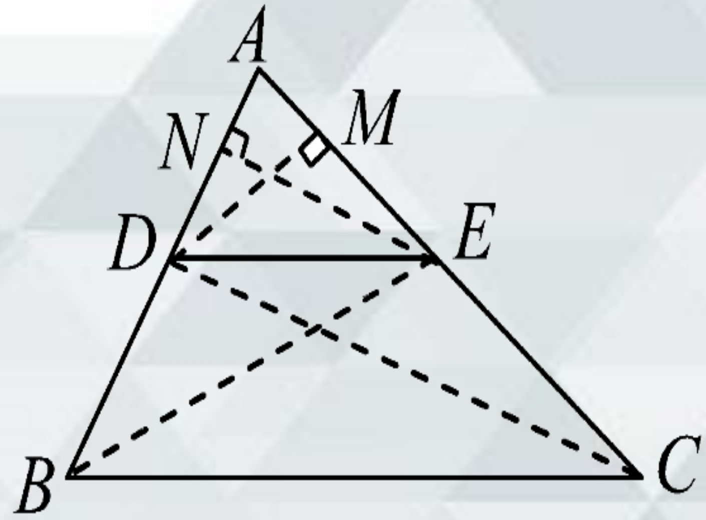


When Thales travelled to Egypt, he was challenged by Egyptians to determine the height of one of several magnificent pyramids that they had constructed. Thales accepted the challenge and used similarity of triangles to determine the same successfully.

PROOF - BASIC PROPORTIONALITY THEOREM

Given: A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$



PROOF - BASIC PROPORTIONALITY THEOREM

Let us join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.

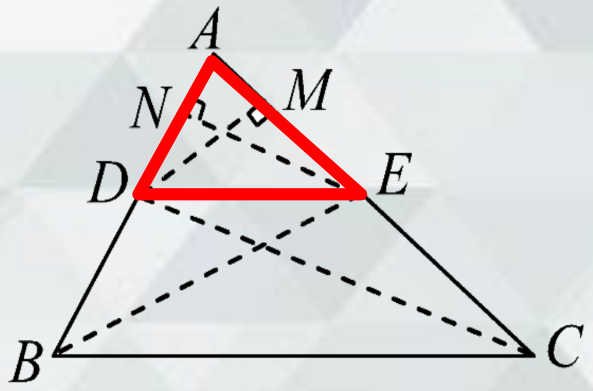
$$\text{ar}(\Delta ADE) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AD \times EN$$

Similarly, $\text{ar}(\Delta ADE) = \frac{1}{2} \times AE \times DM$

$$\text{ar}(\Delta BDE) = \frac{1}{2} \times DB \times EN$$

$$\text{ar}(\Delta DEC) = \frac{1}{2} \times EC \times DM$$

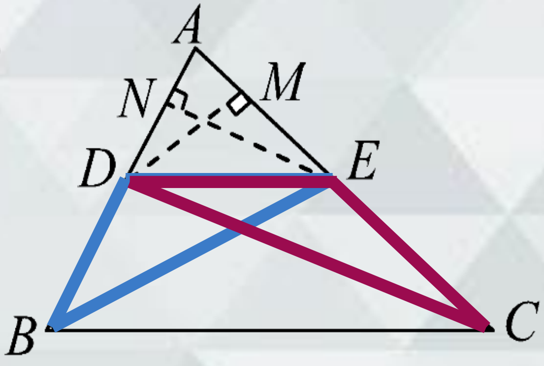


Therefore, $$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}\times AD\times EN}{\frac{1}{2}\times DB\times EN} = \frac{AD}{DB}$$

and $$\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2}\times AE\times DM}{\frac{1}{2}\times EC\times DM} = \frac{AE}{EC}$$

Since, ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE,

$$ar(\Delta BDE) = ar(\Delta DEC)$$



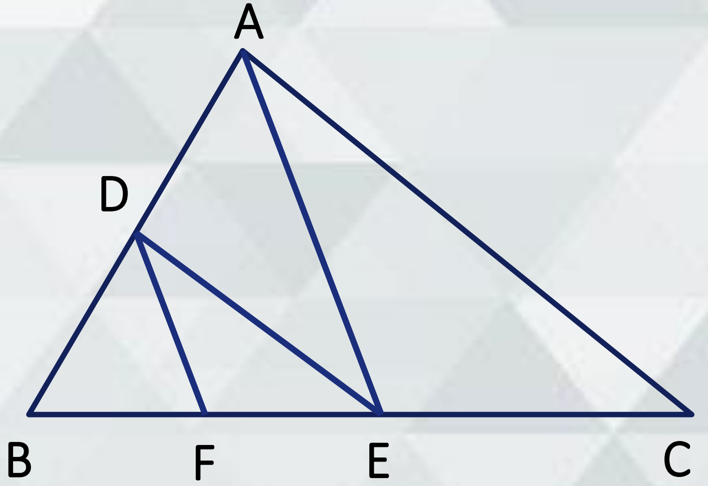
From the above three results, we get

$$\boldsymbol{\frac{AD}{DB} = \frac{AE}{EC}}$$

PROBLEM BASED ON BASIC PROPORTIONALITY THEOREM

In the adjoining figure, DE || AC and DF || AE.

Prove that $\frac{BF}{FE} = \frac{BE}{EC}$



Solution:

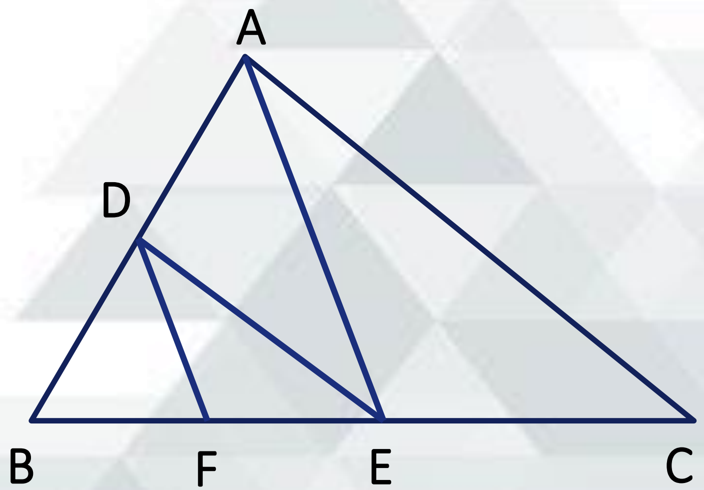
Given: DE || AC and DF || AE

To Prove: $\frac{BF}{FE} = \frac{BE}{EC}$

Consider ΔABE; It is given that DF || AE

Therefore, by using Basic Proportionality Theorem,

$$\frac{BF}{FE} = \frac{BD}{DA} \qquad \text{------------ (1)}$$



Consider ΔABC; It is given that DE || AC

Therefore, by using Basic Proportionality Theorem,

$$\frac{BE}{EC} = \frac{BD}{DA} \qquad \text{------------ (2)}$$

From (1) and (2), we get,

$$\frac{BF}{FE} = \frac{BE}{EC}$$

APPLICATIONS OF SIMILARITY

Here are few applications of similarity

- By analyzing the shadows that make triangles, we can determine the actual height of the objects.
- Used in aerial photography to determine the distance from sky to a particular location on the ground.
- Used in Architecture to aid in design of their work.

THANK YOU