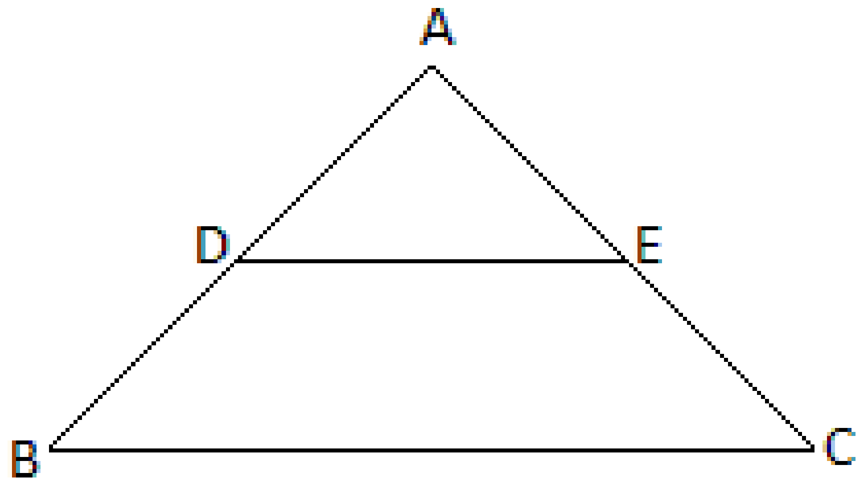
TRIANGLES

MODULE 2

CONTENTS

- Converse of Basic Proportionality Theorem
- Criteria for similarity
  - Angle-Angle similarity

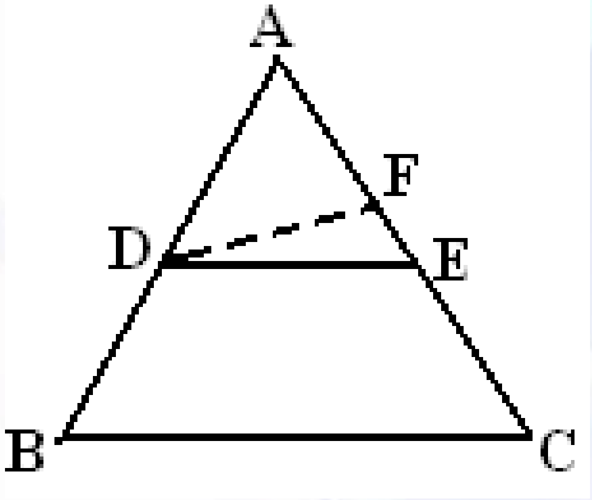
CONVERSE OF BASIC PROPORTIONALITY THEOREM



If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side

PROOF: CONVERSE OF BASIC PROPORTIONALITY THEOREM

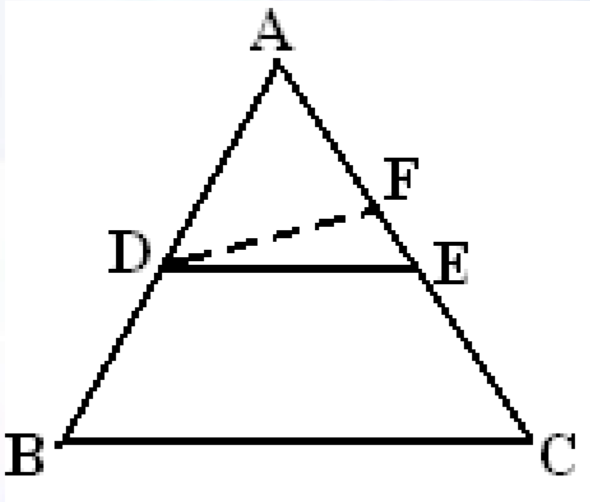
- Given : In ΔABC, $\frac{AD}{DB} = \frac{AE}{EC}$
- To prove : DE || BC
- Construction: Draw a line segment DF such that DF ||BC



Proof

Statement	Reason
In ΔABC, DF || BC	Construction
$\frac{AD}{DB} = \frac{AE}{EC}$ ----------------- (1)	Given
$\frac{AD}{DB} = \frac{AF}{FC}$ ----------------- (2)	Thales theorem (In ΔABC taking F in AC)

Statement	Reason
$\frac{AE}{EC} = \frac{AF}{FC}$	From (1) and (2)
$\frac{AE}{EC} + 1 = \frac{AF}{FC} + 1$ $\frac{AE + EC}{EC} = \frac{AF + FC}{FC}$ $\frac{AC}{EC} = \frac{AC}{FC}$	Adding 1 to both sides
EC = FC Therefore, F coincides with E Thus DE || BC	Cancelling AC on both sides



HENCE PROVED

PROBLEM #1

In the adjoining figure, D and E are the points on AB and AC respectively.

Given :

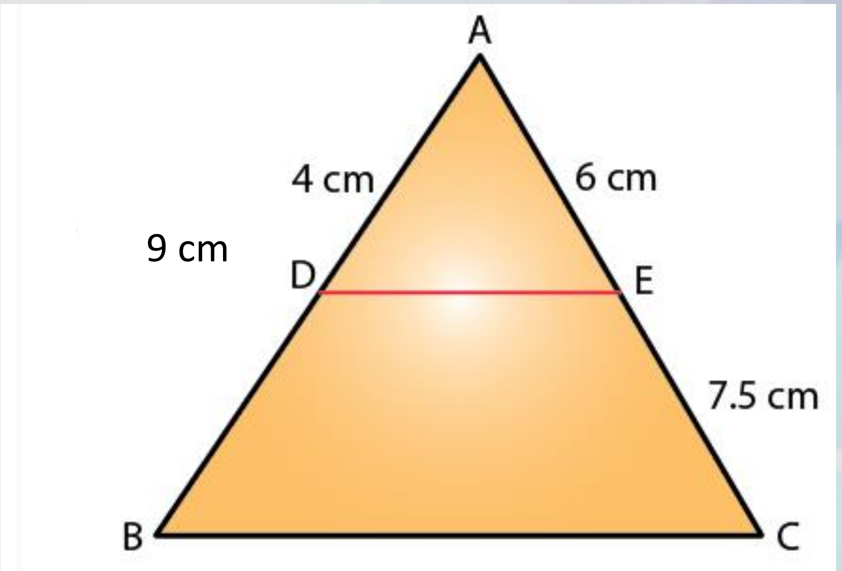
AD = 4cm,

DB = 9cm,

AE = 6cm and

EC = 7.5cm

Check whether, DE || BC.



SOLUTION

In Δ APQ and Δ ABC,

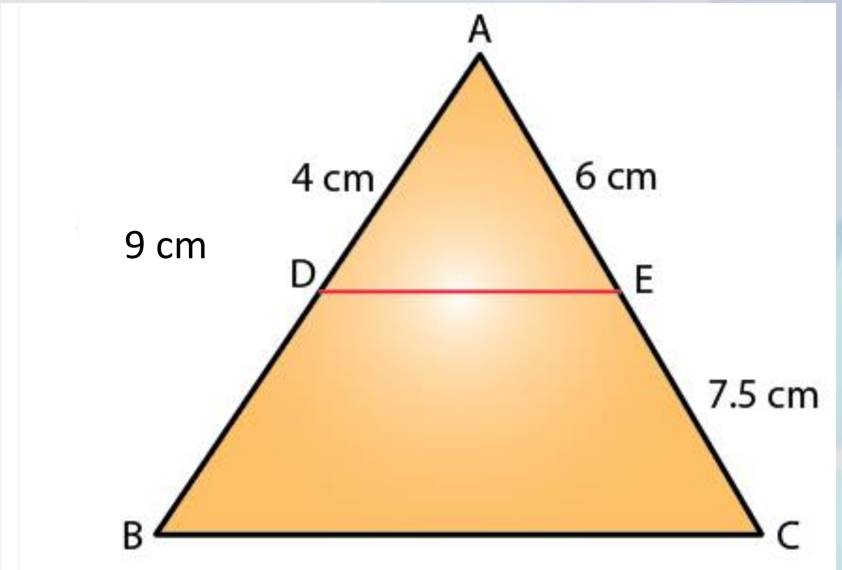
$$\frac{AE}{EC} = \frac{6}{7.5} = \frac{4}{5}$$

BD = AB – AD = 9 – 4 = 5 cm

$$\frac{AD}{BD} = \frac{4}{5}$$

$$\frac{AD}{BD} = \frac{AE}{EC}$$

Therefore, DE || BC by the converse of BPT.



PROBLEM #2

In the adjoining figure, State whether PQ || EF.

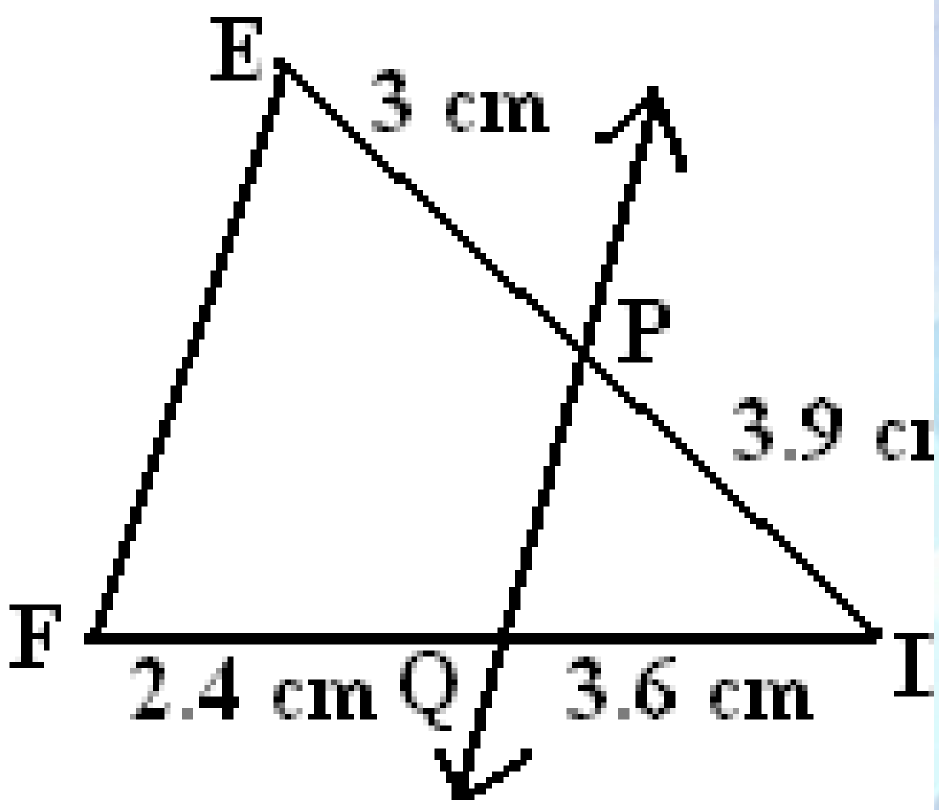
Given,

DP = 3.9cm,

PE = 3cm,

DQ = 3.6cm

and QF = 2.4cm

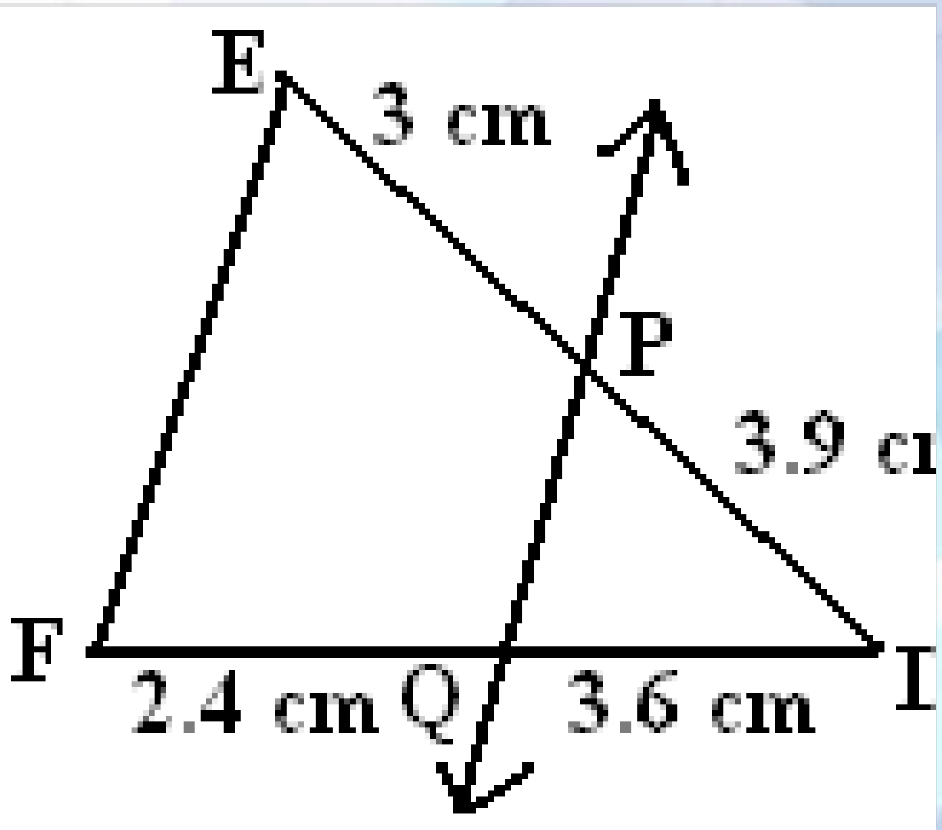


SOLUTION

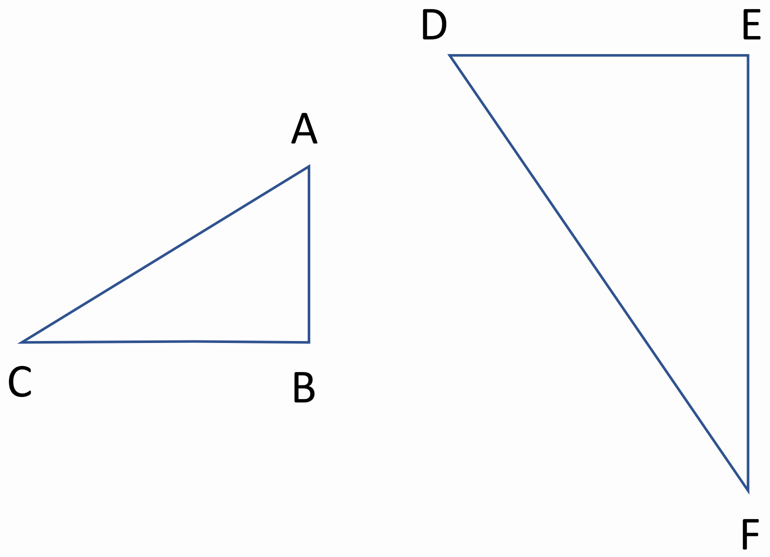
DP / PE = 3.9 / 3 = 13/10

DQ / QF = 3.6 /2.4 = 3/2

So, DP / PE ≠ DQ / QF
⇒ PQ ∦ EF (PQ is not parallel to EF)



SIMILARITY OF TRIANGLES



$\Delta ABC \sim \Delta DEF$

$\Delta\mathrm{ABC}$ and $\Delta\mathrm{DEF}$ are similar to each other.

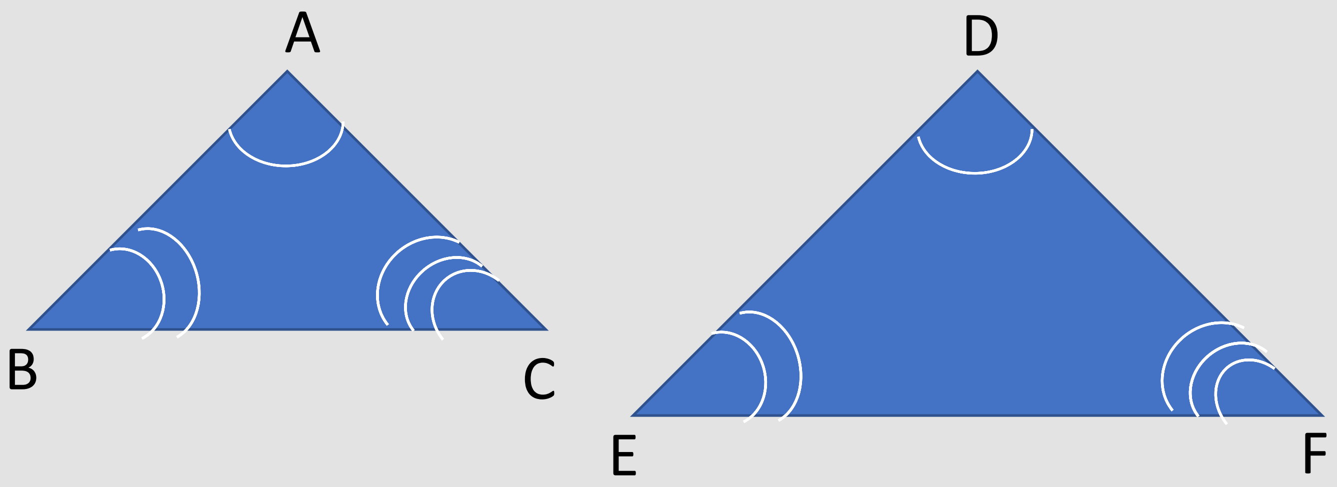
- ∠A = ∠D
- ∠B = ∠E
- ∠C = ∠F
- $\frac{\mathrm{AB}}{\mathrm{DE}} = \frac{\mathrm{BC}}{\mathrm{EF}} = \frac{\mathrm{AC}}{\mathrm{DF}}$

NOTE: As done in the case of congruency of two triangles, the similarity of two triangles should also be expressed symbolically, using correct correspondence of their vertices. For example, for the triangles ABC and DEF, we cannot write Δ ABC ~ Δ EDF or Δ ABC ~ Δ FED. However, we can write Δ BAC ~ Δ EDF

CRITERIA FOR SIMILARITY OF TRIANGLES

- Even if the triangles have six parts, similarity of triangles can be proved by establishing relationship between less number of pairs of corresponding parts of the two triangles.
- Following are the criteria for similarity of triangles:
  - Angle – Angle – Angle similarity
  - Side – Side – Side similarity
  - Side – Angle – Side similarity

Angle – Angle – Angle (AAA) Criteria for Similarity of Triangles



- If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar
- Given: In ΔABC and ΔDEF, ∠A = ∠D, ∠B = ∠E, ∠C = ∠F
- To Prove:

  AB/DE = BC/EF = AC/DF and ΔABC ~ Δ DEF

Proof: AAA Criterion for Similarity of Triangles

Construction: Draw PQ such that DP = AB and DQ = AC

Proof:

In ΔABC and ΔDPQ, ∠A = ∠D, DP = AB and DQ = AC

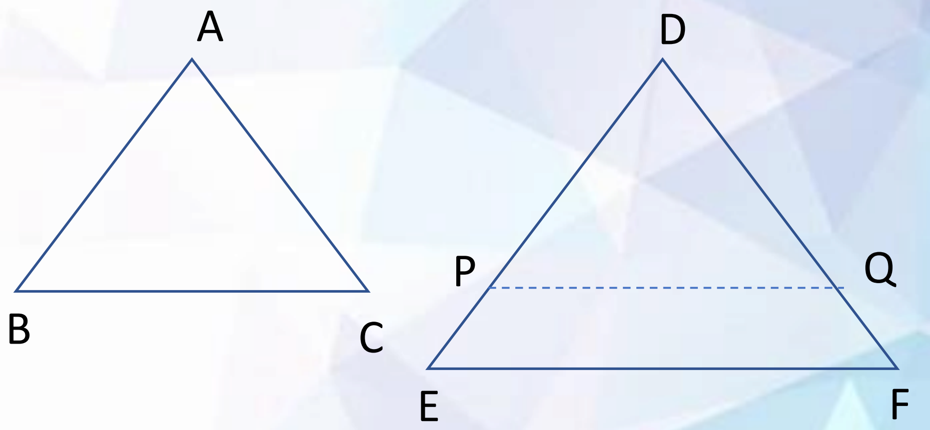
Therefore, ΔABC ≅ ΔDPQ (by SAS congruence criteria)

∠B = ∠DPQ (Corresponding parts of congruent triangles)

Also given that ∠B = ∠E,

∴ ∠E = ∠DPQ

This implies, PQ || EF (Since corresponding angles are equal)



Therefore, $\frac{DP}{PE} = \frac{DQ}{QF}$ (Basic Proportionality Theorem)

This implies $\frac{PE}{DP} = \frac{QF}{DQ}$ (Reciprocal)

$$\frac{PE}{DP} + 1 = \frac{QF}{DQ} + 1$$

$$\frac{DE}{DP} = \frac{DF}{DQ}$$

$$\frac{DP}{DE} = \frac{DQ}{DF} \text{ (Reciprocal)}$$

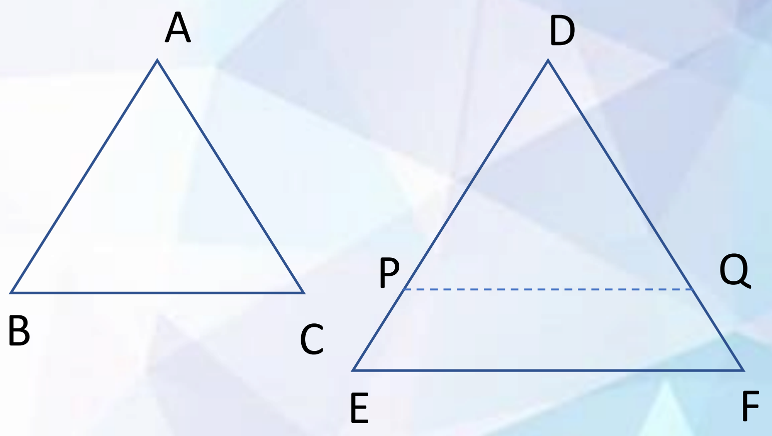
i.e., $\frac{AB}{DE} = \frac{AC}{DF}$ (By construction DP = AB, DQ = AC)

Similarly when P’Q’ is constructed so that AB = EQ’ and BC = EP’ it can be proved that $\frac{AB}{DE} = \frac{BC}{EF}$

Thus $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

In ΔABC and ΔDEF, given that the corresponding angles are equal and we have proved that the corresponding sides are in the same ratio.

Hence ΔABC ~ Δ DEF



AA Criterion for Similarity of Triangles

- If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal. Therefore, AAA similarity criterion can also be stated as follows:
- If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. This may be referred to as the AA similarity criterion for two triangles

In triangles ABC and DEF

Case 1:

- ∠A = ∠D = 30°
- ∠B = ∠E = 60°
- ∠C = ∠F = 90°

ΔABC ~ Δ DEF

SIMILAR

Case 2:

- ∠A = ∠D = 40°
- ∠B = 50°
- ∠E = 90°

ΔABC ~ Δ DFE

SIMILAR

Case 3:

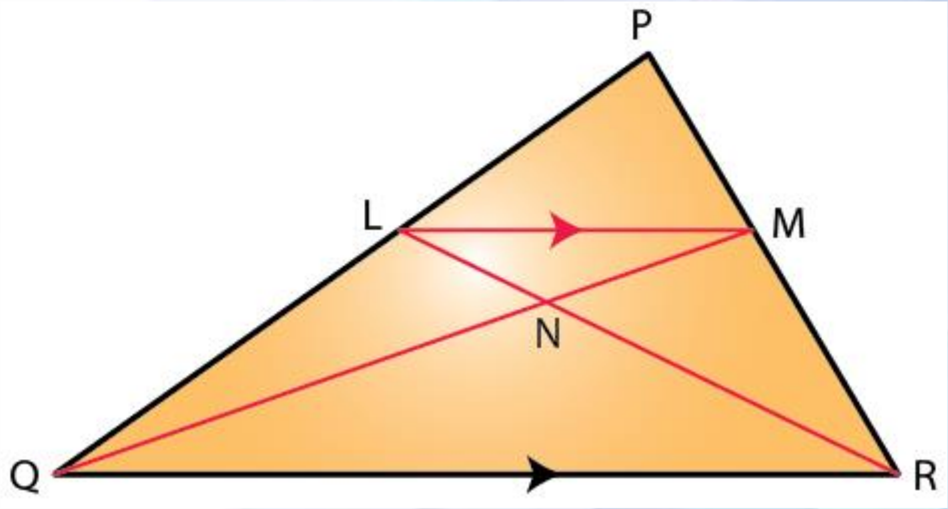
- ∠A ≠ ∠D,
- ∠B = ∠E,
- ∠C ≠ ∠F

NOT SIMILAR

SOLVED EXAMPLE

In the given triangle PQR, LM is parallel to QR and PM: MR = 3: 4. Calculate the value of ratio PL/PQ and LM/QR

Solution:



In Δ PLM and Δ PQR,

As LM || QR, corresponding angles are equal.

∠PLM = ∠PQR

∠PML = ∠PRQ

Hence, ΔPLM ~ ΔPQR by AA criterion for similarity.

So, we have

PM/ PR = LM/ QR

3/7 = LM/QR [Since, PM/MR = ¾ ⇒ PM/PR = 3/7]

And, by BPT we have

PL/LQ = PM/MR = ¾

LQ/PL = 4/3

1 + (LQ/PL) = 1 + 4/3

(PL + LQ)/ PL = (3 + 4)/3

PQ/PL = 7/3

Hence, PL/PQ = 3/7

Thank You