TRIANGLES

MODULE 3

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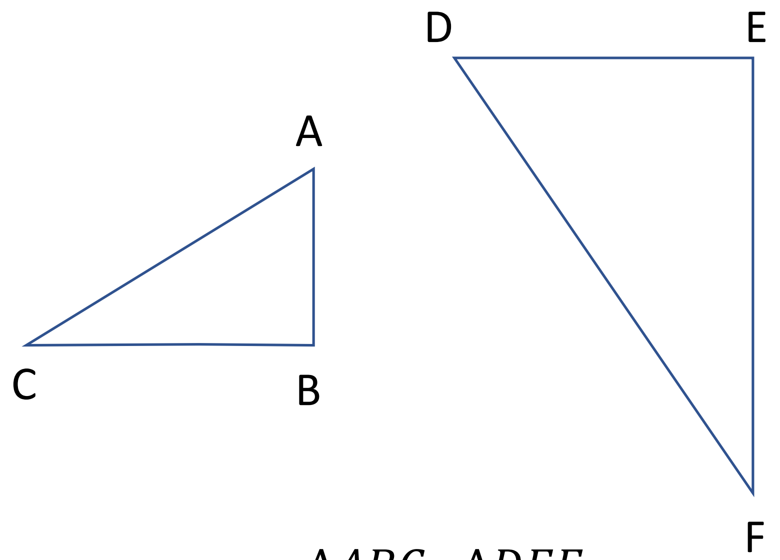
PROPERTIES OF SIMILAR TRIANGLES



$\Delta ABC \sim \Delta DEF$

ΔABC and ΔDEF are similar to each other.

- ∠A = ∠D
- ∠B = ∠E
- ∠C = ∠F
- $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

CRITERIA FOR SIMILARITY OF TRIANGLES

1. Angle – Angle – Angle Similarity Criterion
   - If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar

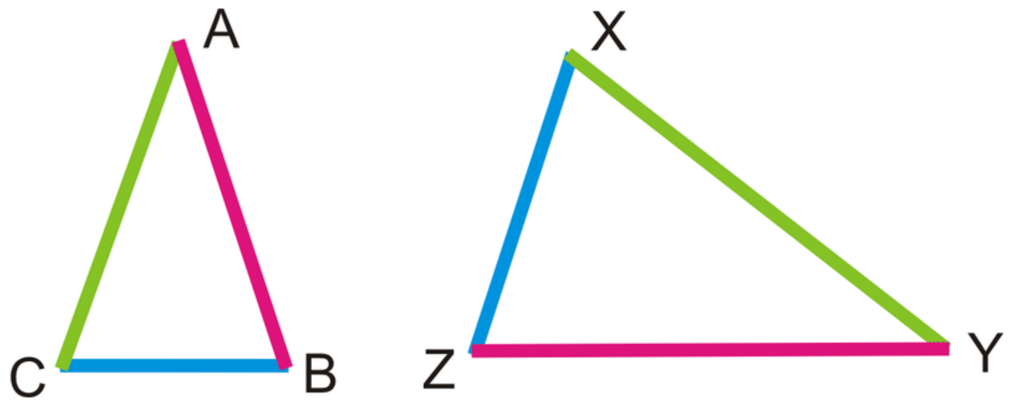
   Corollary: Angle – Angle Similarity Criterion
   - If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

2. Side – Side – Side Similarity Criterion
3. Side – Angle – Side Similarity Criterion

Side – Side – Side Similarity Criterion

STATEMENT:

If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.



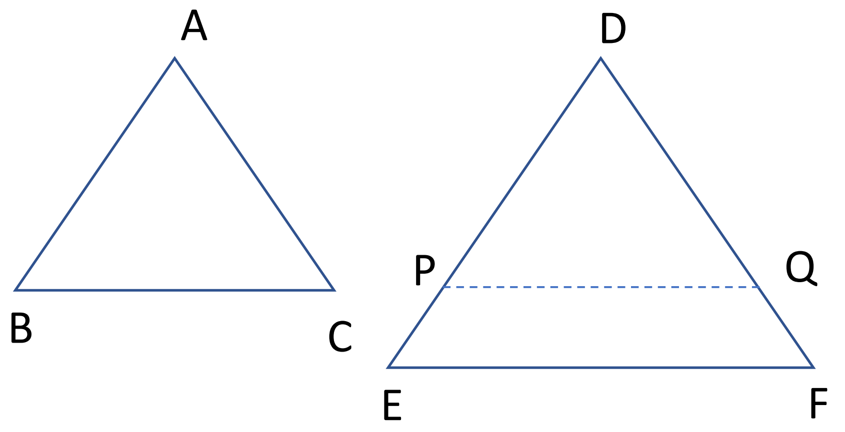
$$\frac{AB}{YZ} = \frac{BC}{XZ} = \frac{AC}{XY}$$

ΔABC ~ Δ YZX

Proof: SSS Criterion for Similarity of Triangles

- Given: In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$
- To Prove:

  ∠A = ∠D, ∠B = ∠E, ∠C = ∠F and ΔABC ~ Δ DEF
- Construction: Draw PQ such that DP = AB and DQ = AC



Proof:

In ΔABC and ΔDPQ, $\frac{AB}{DE} = \frac{AC}{DF}$, DP = AB and DQ = AC

Therefore, $\frac{DP}{DE} = \frac{DQ}{DF} \Longrightarrow \frac{DE}{DP} = \frac{DF}{DQ}$

Subtracting 1 from both sides and taking reciprocal, we get $\frac{DP}{PE} = \frac{DQ}{QF}$

Thus PQ || EF (By converse of Basic Proportionality Theorem)

∠DPQ = ∠E and ∠DQP = ∠F

ΔDPQ ~ ΔDEF (by AA similarity criteria)

Therefore, $\frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$

Given the equality of ratios, $\frac{DP}{DE} = \frac{DQ}{DF} = \frac{BC}{EF}$

∴ BC = PQ

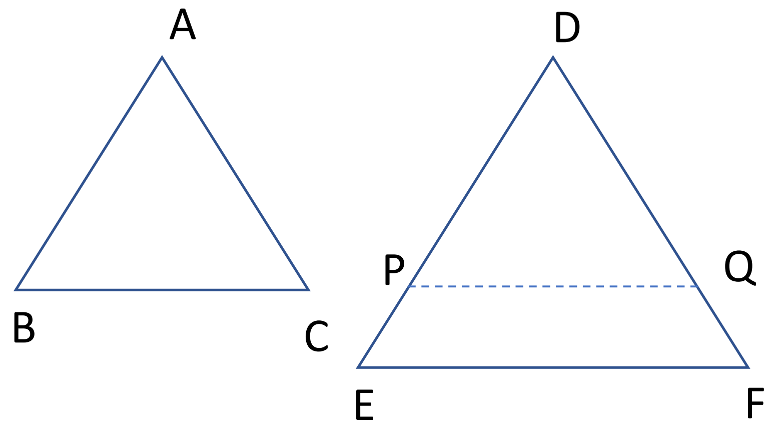
ΔABC ≅ Δ DPQ (SSS congruence condition)

By corresponding parts of congruent triangles, ∠A = ∠D

Also, ∠B = ∠E and ∠C = ∠F

In ΔABC and ΔDEF, given that the corresponding sides are in the same ratio and we have proved that the corresponding angles are equal.

Hence ΔABC ~ Δ DEF



SOLVED EXAMPLE

Are triangles VZW and VXY similar? If yes, mention the similarity criteria and similarity relationship.

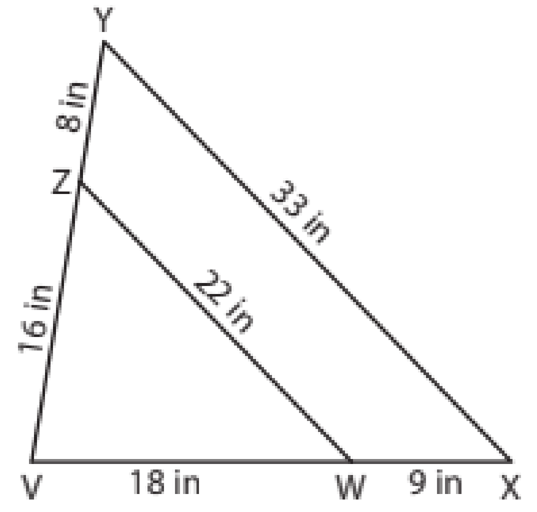
Solution:

$$\frac{VZ}{VY} = \frac{16}{16+8} = \frac{2}{3}$$

$$\frac{VW}{VX} = \frac{18}{18+9} = \frac{2}{3}$$

$$\frac{WZ}{XY} = \frac{22}{33} = \frac{2}{3}$$

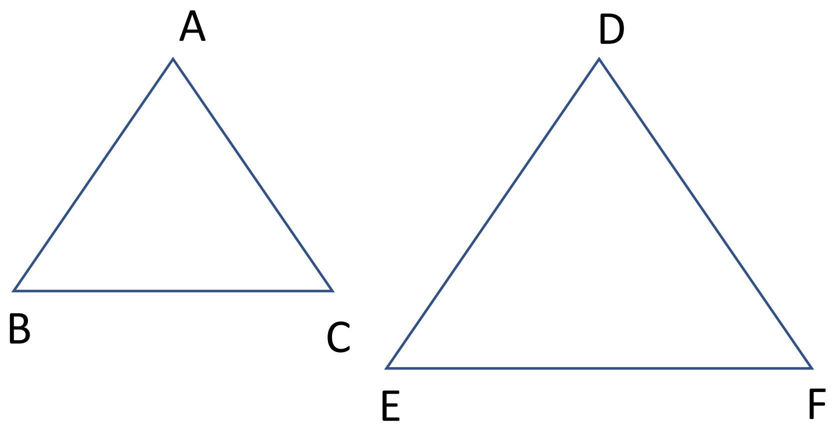
Since, $\frac{VZ}{VY} = \frac{VW}{VX} = \frac{WZ}{XY}$, $\Delta \boldsymbol{VZW} \sim \Delta \boldsymbol{VYX}$ by **SSS similarity criterion**



Side – Angle – Side Similarity Criterion

STATEMENT:

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

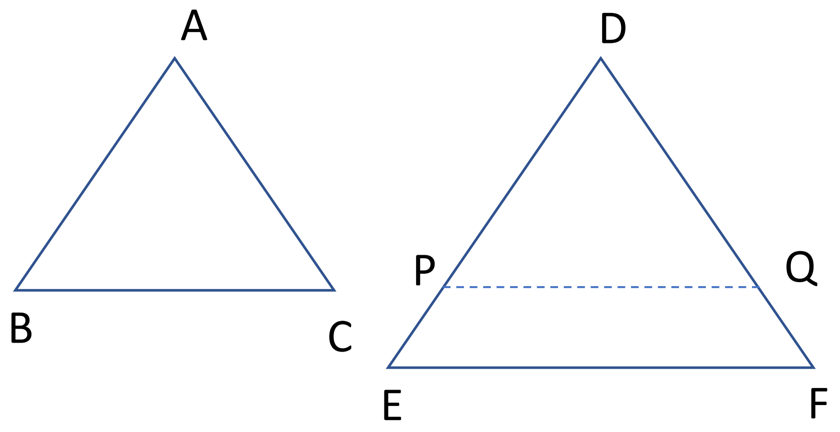


$$\frac{AB}{DE} = \frac{AC}{DF}, \angle A = \angle D$$

ΔABC ~ Δ DEF

Proof: SAS Criterion for Similarity of Triangles

- Given: In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$
- To Prove: ΔABC ~ Δ DEF
- Construction: Draw PQ such that DP = AB and DQ = AC



Proof:

In ΔABC and ΔDPQ, $\frac{AB}{DE} = \frac{AC}{DF}$, DP = AB and DQ = AC

Therefore, $\frac{DP}{DE} = \frac{DQ}{DF} \Longrightarrow \frac{DE}{DP} = \frac{DF}{DQ}$

Subtracting 1 from both sides and taking reciprocal, we get $\frac{DP}{PE} = \frac{DQ}{QF}$

Thus PQ || EF (By converse of Basic Proportionality Theorem)

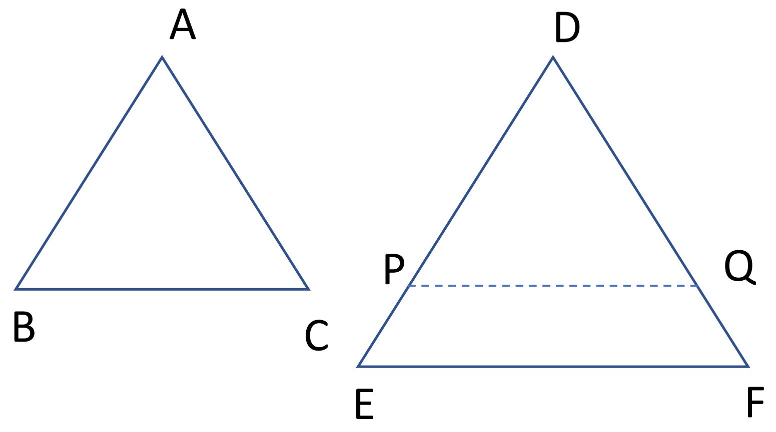
∠DPQ = ∠E and ∠DQP = ∠F

ΔABC ≅ ΔDPQ (by SAS congruence criteria)

∠B = ∠DPQ and ∠DQP = ∠C (Corresponding parts of congruent triangles)

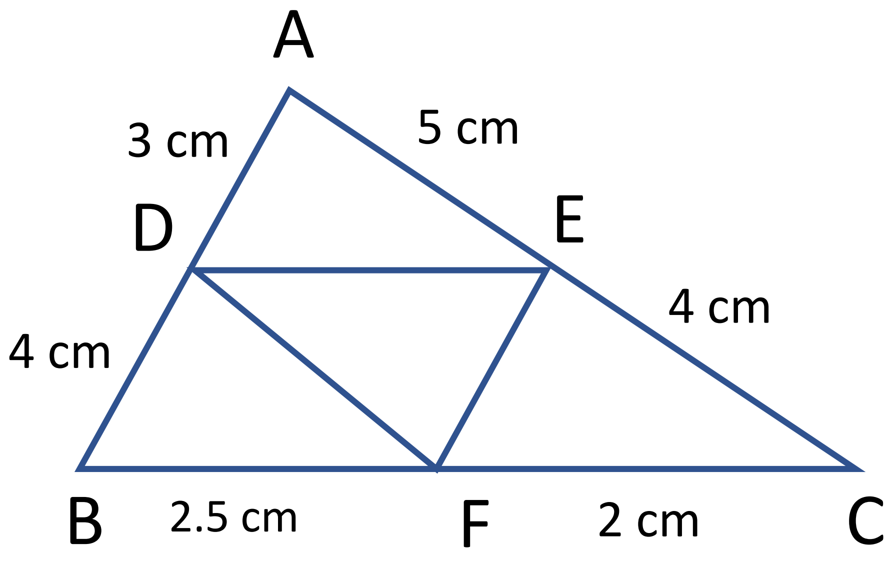
∴ ∠B = ∠E and ∠C = ∠F

Hence, by AAA similarity criterion, ΔABC ~ Δ DEF



SOLVED EXAMPLE

In the adjoining fig., AD = 3cm, AE = 5cm, BD = 4cm, CE = 4cm, CF = 2cm, BF = 2.5cm, then find the pair of parallel lines and hence the lengths of the same.



Solution:

$\frac{EC}{EA} = \frac{4}{5}$ and

$\frac{CF}{FB} = \frac{2}{2.5} = \frac{4}{5}$

$\implies \frac{EC}{EA} = \frac{CF}{FB}$

Therefore by converse of Basic Proportionality Theorem,
EF||AB

Also, $\frac{CE}{CA} = \frac{4}{4+5} = \frac{4}{9}$

$\frac{CF}{CB} = \frac{2}{2+2.5} = \frac{2}{4.5} = \frac{4}{9}$

Therefore, By SAS Similarity

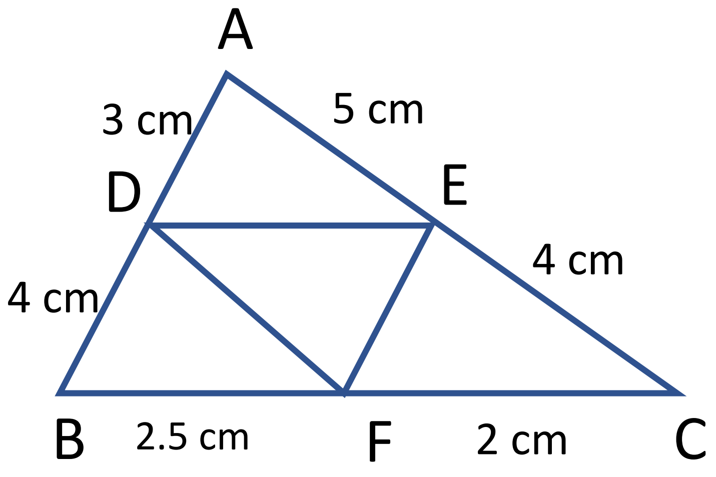
ΔCFE ~ ΔCBA

$\Longrightarrow \frac{EF}{AB} = \frac{CE}{CA}$

$\Longrightarrow \frac{EF}{7} = \frac{4}{9}$

EF = $\frac{28}{9}$

AB =7cm



THANK YOU