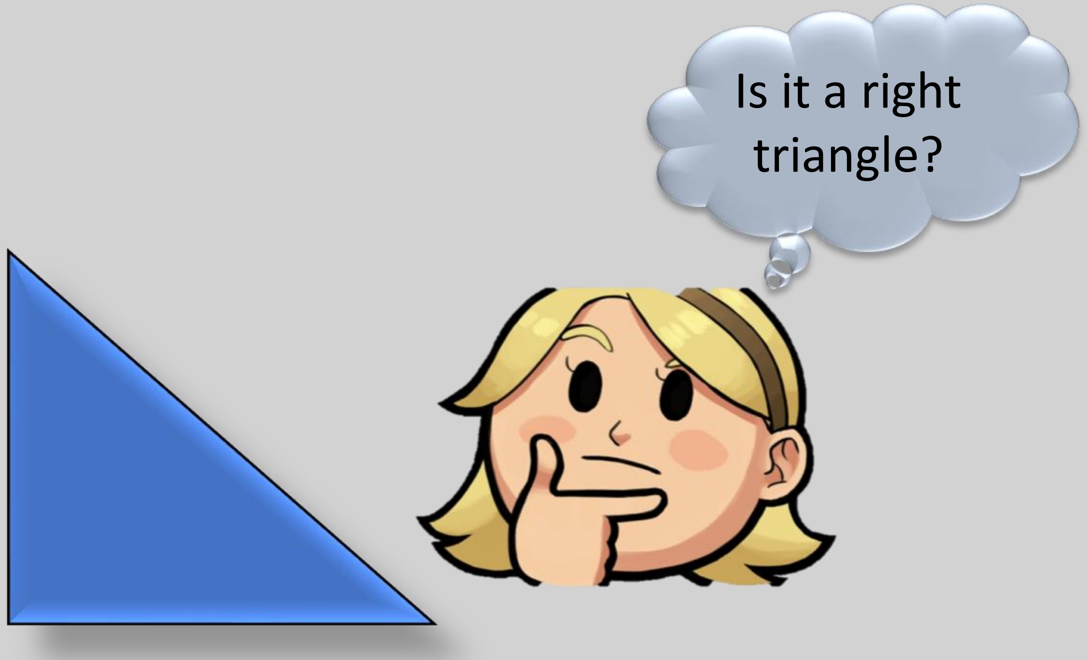
TRIANGLES

MODULE 5

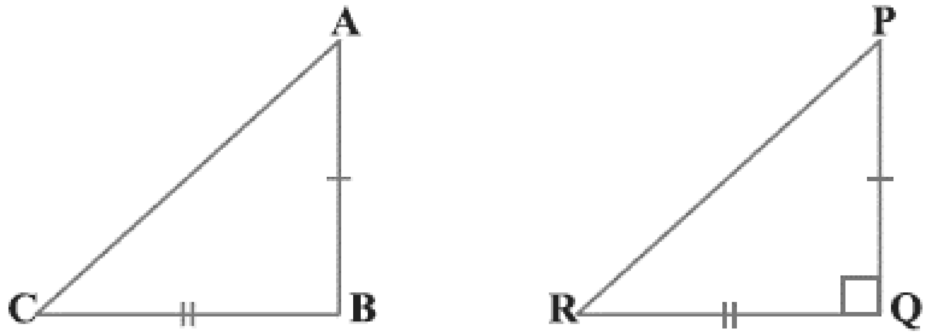
CONVERSE OF PYTHAGORAS THEOREM

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.



PROOF OF CONVERSE OF PYTHAGORAS THEOREM

- Given: A triangle ABC in which $AC^2 = AB^2 + BC^2$
- To Prove: ∠B = 90°
- Construction: Δ PQR right angled at Q such that PQ = AB and QR = BC



Consider Δ PQR

$PR^2 = PQ^2 + QR^2$ (By Pythagoras Theorem as ∠Q = 90°)

By construction PQ = AB and QR = BC

$\Longrightarrow PR^2 = AB^2 + BC^2$ ------ (1)

We are given that $AC^2 = AB^2 + BC^2$ ------ (2)

From (1) and (2)

PR = AC

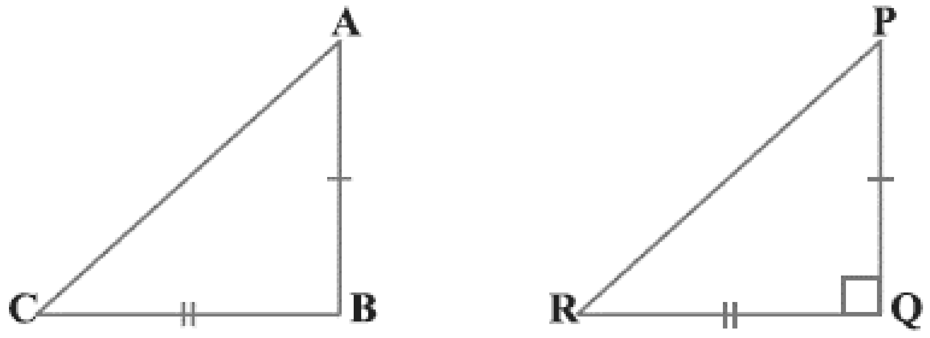
Also in Δ ABC and Δ PQR,

AB = PQ (By construction)

BC = QR (By construction)

$\Longrightarrow \Delta ABC \cong \Delta PQR$ (By SSS congruence condition)

$\Longrightarrow \angle B = \angle Q = 90°$ (Corresponding parts of congruent triangles)



SOLVED EXAMPLE

Consider a triangle whose sides are 5cm, 12cm and 13cm. Is this a right triangle or not?

Solution:

We are given that,

AB = 5cm

BC = 12cm

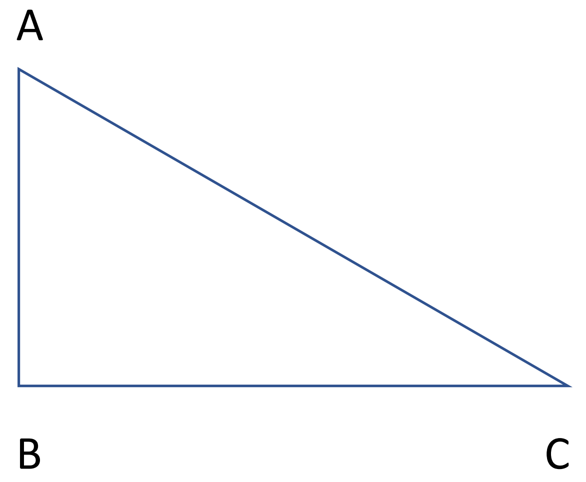
AC = 13cm

$AC^2 = 13^2 = 169$

$AB^2 + BC^2 = 5^2 + 12^2 = 25 + 144 = 169$

$\Longrightarrow AC^2 = AB^2 + BC^2$

Therefore, by converse of Pythagoras theorem the given triangle ABC is a right triangle.



SUMMARY

Two polygons of the same number of sides are similar, if

- their corresponding angles are equal and
- their corresponding sides are in the same ratio (i.e., proportion)

Basic Proportionality theorem

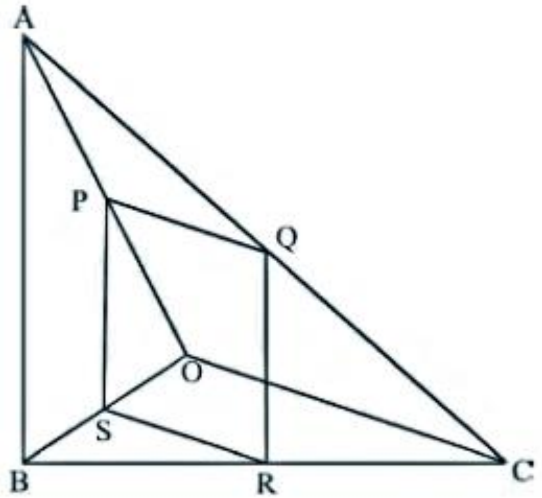
If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio

Converse of Basic Proportionality theorem

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

SOLVED EXAMPLE

In fig, if PQRS is a parallelogram and AB||PS, then prove that OC||SR



Solution:

In ΔABO and ΔPSO,

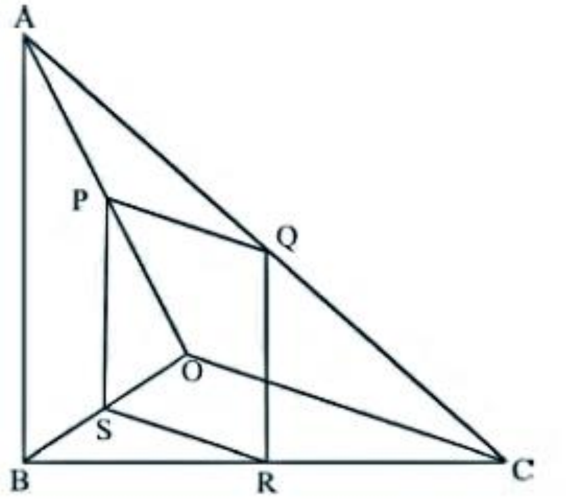
$\angle AOB = \angle POS$ (common angle)

$\angle ABO = \angle PSO$ (Since AB||PS, corresponding angles are equal)

$\Longrightarrow \Delta ABO \sim \Delta PSO$ by AA similarity criterion

$\frac{OP}{OA} = \frac{PS}{AB}$ (Corresponding sides of similar triangles are in the same ratio)

$\frac{OP}{OA} = \frac{QR}{AB}$ (Since, PQRS is a parallelogram, QR = PS) ------------- (1)



Since, PQRS is a parallelogram, QR||PS and we are given that AB||PS

$\Longrightarrow$ QR||AB

$\Longrightarrow \angle ABC = \angle QRC$ (corresponding angles are equal)

$\Longrightarrow \angle ACB = \angle QCR$ (Common)

$\Longrightarrow \Delta ABC \sim \Delta QRC$ by AA similarity criterion

$\frac{QR}{AB} = \frac{QC}{AC}$ (Corresponding sides of similar triangles are in the same ratio) ------------ (2)

From (1) and (2)

$\frac{OP}{OA} = \frac{QC}{AC}$

$\Longrightarrow$ PQ||OC (By converse of Basic Proportionality Theorem)

$\Longrightarrow$ SR||OC (Since PQRS is a parallelogram, PQ||SR)

SIMILARITY CRITERIA OF TRIANGLES

- If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (AA similarity criterion).
- If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (SSS similarity criterion).
- If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar (SAS similarity criterion).
- **Note:** If in two right triangles, hypotenuse and one side of one triangle are proportional to the hypotenuse and one side of the other triangle, then the two triangles are similar. This may be referred to as the RHS Similarity Criterion.

SOLVED EXAMPLE

In Δ ABC, AB=BC and D is a point on side AC, such that BC^2 = AC x CD. Prove that BD = BC.

Given:

AB = BC in ΔABC and D is a point on BC.

To Prove: BD = BC

Proof: BC^2= AC x CD

$\Longrightarrow \frac{BC}{AC} = \frac{DC}{BC}$

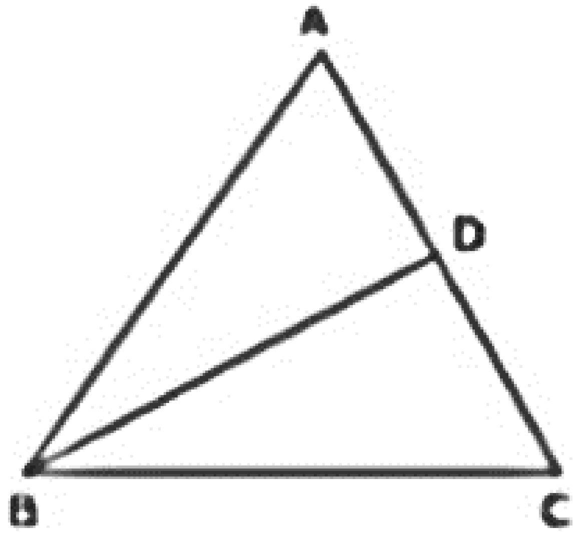
In ΔABC and ΔBDC

$\Longrightarrow \frac{BC}{CA} = \frac{DC}{CB}$ and $\angle C$ = $\angle C$ [Common]

$\Delta ADC \sim \Delta BDC$ [SAS Similarity]

$\Longrightarrow \frac{AB}{BD} = \frac{AC}{BC} \Longrightarrow \frac{AC}{BD} = \frac{AC}{BC}$ [because AB= AC]

$\Longrightarrow$ BD = BC



- AREAS OF SIMILAR TRIANGLES

  The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

- PYTHAGORAS THEOREM

  In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides

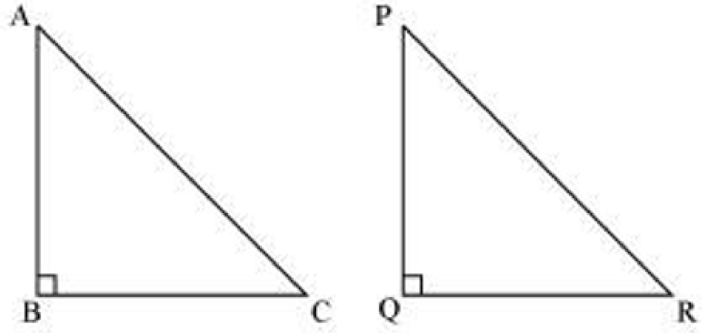
- CONVERSE OF PYTHAGORAS THEOREM

  If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

SOLVED EXAMPLE

ΔABC and ΔPQR are two isosceles right triangles such that ar(ΔABC) : ar(ΔPQR) = 25:18. The length of the hypotenuse of ΔABC is 30cm. What is the perimeter of ΔPQR?

Let ΔABC and ΔPQR be the given two isosceles right triangles where $\angle B = 90°$ and $\angle Q = 90°$.



$\angle B = \angle Q = 90°$

$\angle A = \angle P = 45°$

$\angle C = \angle R = 45°$

Therefore ΔABC ∼ ΔPQR [By AAA Similarity criterion]

The ratios of the areas of similar triangles is equal to the ratio of the squares of their corresponding sides.

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AC}{PR}\right)^2$

$\frac{25}{18} = \frac{30cm^2}{PR^2}$

PR^2 = $(\frac{900 * 18}{25})\ cm^2$

Applying Pythagoras theorem to the right triangle PQR,

PR^2 = PQ^2 + QR^2

$\Longrightarrow PQ^2$ + PQ^2 = 648 cm^2 [ΔPQR is isosceles where $\angle Q = 90°$

$\Longrightarrow 2PQ^2$ = 648 cm^2

$\Longrightarrow PQ^2$ = 324 cm^2

$\Longrightarrow$ PQ = 18 cm

$\Longrightarrow$ QR = 18 cm

Perimeter of ΔPQR = PQ +QR + PR = (18 + 18 + 18 $\sqrt{2}$) cm = (36 + 18 $\sqrt{2}$) cm

= 18(2 + $\sqrt{2}$) cm