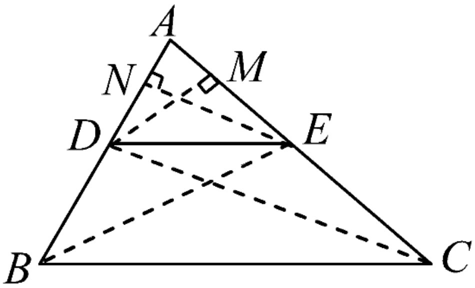
TRIANGLES – 1/5

1. A ***polygon*** is any simple closed curve made up of only line segments.
2. Congruent figures: Two figures are said to be ***congruent*** if they have same shape and same size.
3. Similar figures: Two figures having same shape and not necessarily the same size are called ***similar figures***.
   - ❖ Two polygons with the same number of sides are similar if
     i. Their corresponding angles are equal, and
     ii. Their corresponding sides are in the same ratio (or proportion)
4. ***Congruence*** is a particular case of similarity. In both the cases, three angles of one triangle are equal to the three corresponding angles of the other triangle. But in ***congruent triangles***, the corresponding sides are equal while in similar triangles the corresponding sides are proportional.
   - ❖ If ΔABC and ΔPQR are similar, then it can be represented as $\Delta ABC \sim \Delta PQR$.
5. ***Two triangles are similar***, if
   i. Their corresponding angles are equal, and
   ii. Their corresponding sides are in the same ratio (or proportion)
6. If the corresponding angles of the two triangles are equal, they are called ***equiangular triangles.***
7. The ratio of any two corresponding sides in two equiangular triangles is always the same.
8. ***Basic Proportionality Theorem*** (Thales Theorem):
   If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



Given: A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

To prove: $\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$

Proof:Let us join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.

$$\text{ar}(\Delta\text{ADE}) = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \text{AD} \times \text{EN}$$

Similarly, $\text{ar}(\Delta\text{BDE}) = \frac{1}{2} \times \text{DB} \times \text{EN}$

$$\text{ar}(\Delta\text{ADE}) = \frac{1}{2} \times \text{AE} \times \text{DM}$$

$$\text{ar}(\Delta\text{DEC}) = \frac{1}{2} \times \text{EC} \times \text{DM}$$

Therefore,

$$\frac{\text{ar}(\Delta\text{ADE})}{\text{ar}(\Delta\text{BDE})} = \frac{\frac{1}{2} \times \text{AD} \times \text{EN}}{\frac{1}{2} \times \text{DB} \times \text{EN}} = \frac{\text{AD}}{\text{DB}}$$

$$\frac{\text{ar}(\Delta\text{ADE})}{\text{ar}(\Delta\text{DEC})} = \frac{\frac{1}{2} \times \text{AE} \times \text{DM}}{\frac{1}{2} \times \text{EC} \times \text{DM}} = \frac{\text{AE}}{\text{EC}}$$

Since, ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE,

$$\text{ar}(\Delta\text{BDE}) = \text{ar}(\Delta\text{DEC})$$

From the above three results, we get

$$\boldsymbol{\frac{AD}{DB} = \frac{AE}{EC}}$$

Solved Example*:* In ΔABC, if DE || BC, AD = x, DB = x - 2, and EC = x - 1 then find the lengths of the sides AB and AC.

Solution In ΔABC, we have DE || BC.

By Thales theorem, we have $\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$

Therefore, $\frac{\text{x}}{\text{x}-2} = \frac{\text{x}+2}{\text{x}-1}$

This gives x(x-1) = (x-2) (x+2)

Hence, $x^2 - x = x^2 - 4$

So, x = 4

When $x = 4$, $AD = 4$, $DB = x - 2 = 2$, $AE = x + 2 = 6$, $EC = x - 1 = 3$

Hence, $AB = AD + DB = 4 + 2 = 6$, $AC = AE + EC = 6 + 3 = 9$

Therefore, $AB = 6$, $AC = 9$.