TRIANGLES – MODULE 2/5

Converse of Basic Proportionality Theorem:

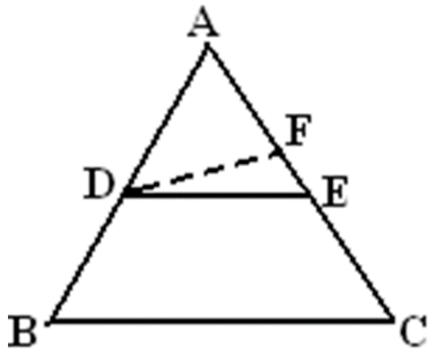
Statement: If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Given: A ΔABC and a line segment intersecting AB in D and AC in E, such that $\frac{AD}{DB} = \frac{AE}{EC}$.

To Prove: DE ∥ BC.

Construction: Draw a line segment DF such that DF || BC

Proof: Assume DE not parallel to BC. Then, there must be another line through D parallel to BC.



Let DF ∥ BC. Then, by Basic Proportionality Theorem, we've

$\frac{AD}{DB} = \frac{AF}{FC}$ --------- (i)

But, Since $\frac{AD}{DB} = \frac{AE}{EC}$ --------- (ii)

From (i) and (ii), we have

$$\frac{AE}{EC} = \frac{AF}{FC}$$

$$\frac{AE}{EC} + 1 = \frac{AF}{FC} + 1$$

$$\frac{AE + EC}{EC} = \frac{AF + FC}{FC}$$

$$\frac{AC}{EC} = \frac{AC}{FC}$$

$$FC = EC$$

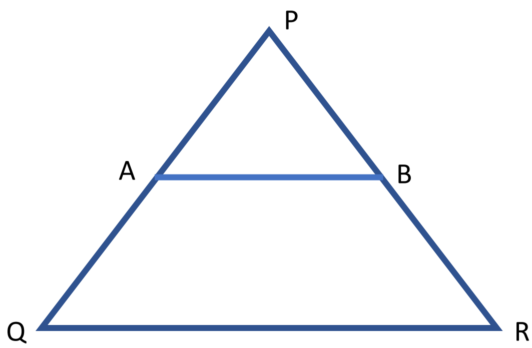
Therefore, F coincides with E

Thus DE || BC

*** Note: This proof is not for examination point of view.

Solved Example:

A and B are respectively the points on the sides PQ and PR of ΔPQR such that PQ = 12.5cm, PA = 5cm, BR = 6cm and PB = 4cm. Is AB || QR?



Solution:

By converse of Basic Proportionality theorem, AB||QR if AB divides PQ and PR in the same ratio. i.e, if $\frac{PA}{AQ} = \frac{PB}{BR}$

QA = QP – PA = 12.5 cm – 5 cm = 7.5 cm

$$\frac{PA}{AQ} = \frac{5}{7.5} = \frac{2}{3}$$

$$\frac{PB}{BR} = \frac{4}{6} = \frac{2}{3}$$

Therefore, we get $\frac{PA}{AQ} = \frac{PB}{BR}$. This implies AB || QR

Criteria for Similarity of Triangles

We have already stated that two triangles are similar if

(i) their corresponding angles are equal.

(ii) their corresponding sides are in the same ratio(proportional).

So, in Δ ABC and ΔDEF, if

- ∠A = ∠D
- ∠B = ∠E
- ∠C = ∠F and
- $\frac{\mathrm{AB}}{\mathrm{DE}} = \frac{\mathrm{BC}}{\mathrm{EF}} = \frac{\mathrm{AC}}{\mathrm{DF}}$

then two triangles Δ ABC and Δ DEF are similar.

i.e., Δ ABC ∼ Δ DEF.

AAA Similarity: If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportional) and hence the two triangles are similar.

Corollary: (AA Similarity) If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.