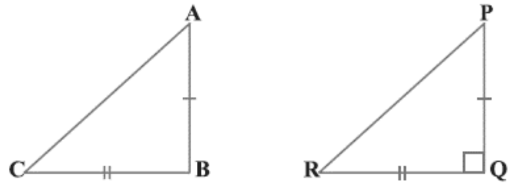
TRIANGLES – MODULE 5/5

CONVERSE OF PYTHAGORAS THEOREM:

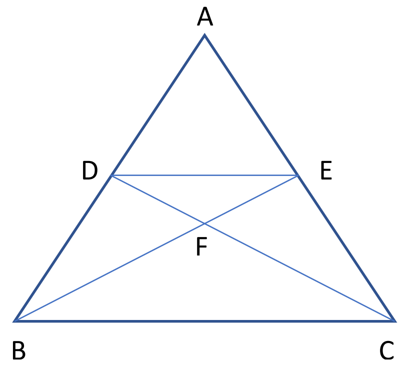
In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Given: A triangle ABC in which $AC^2 = AB^2 + BC^2$
To Prove: $\angle B = 90°$
Construction: Δ PQR right angled at Q such that PQ = AB and QR = BC
Proof:
Consider Δ PQR
$PR^2 = PQ^2 + QR^2$ (By Pythagoras Theorem as $\angle Q = 90°$)
By construction PQ = AB and QR = BC
$\Longrightarrow PR^2 = AB^2 + BC^2$ ------ (1)
We are given that $AC^2 = AB^2 + BC^2$ ------ (2)
From (1) and (2)
PR = AC
Also in Δ ABC and Δ PQR,
AB = PQ (By construction)
BC = QR (By construction)
$\Longrightarrow \Delta ABC \cong \Delta PQR$ (By SSS congruence condition)
$\Longrightarrow \angle B = \angle Q = 90°$ (Corresponding parts of congruent triangles)



SOLVED EXAMPLES

1) In the given figure, DE || BC and AD:DB = 5:4, find $\frac{ar(\Delta DFE)}{ar(\Delta CFB)}$



Solution:

Consider ΔADE and ΔABC
$\angle$A is common
$\angle ADE = \angle ABC$ (Since DE || BC and corresponding angles are equal)
$\Longrightarrow \Delta ADE \sim \Delta ABC$ (By AA similarity criterion)
$\Longrightarrow \frac{AD}{AB} = \frac{DE}{BC}$ (Corresponding parts of similar triangles) -------- (1)

Consider ΔDFE and ΔCFB
$\angle EDF = \angle FCB$ and $\angle DEF = \angle FBC$ (Since DE || BC and alternate interior angles are equal)
$\Longrightarrow \Delta DFE \sim \Delta CFB$ (By AA similarity criterion)

Therefore, the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

From (1), $\frac{ar(\Delta DFE)}{ar(\Delta CFB)} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{AD}{AB}\right)^2$ -------- (2)

$\frac{AD}{DB} = \frac{5}{4} \Rightarrow \frac{DB}{AD} = \frac{4}{5}$

$\Rightarrow \frac{DB}{AD} + 1 = \frac{DB+AD}{AD} = \frac{AB}{AD} \Rightarrow \frac{AD}{AB} = \frac{1}{\frac{4}{5}+1} = \frac{5}{9}$

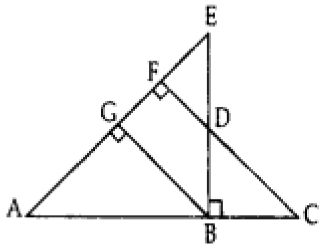
From (2)

$\Rightarrow \frac{ar(\Delta DFE)}{ar(\Delta CFB)} = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$

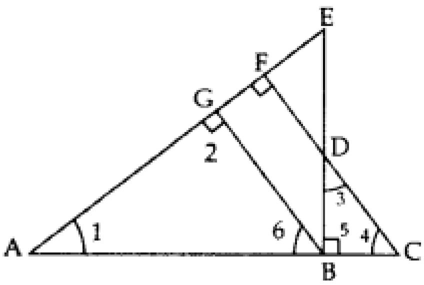
2) In given figure, EB ⊥ AC, BG ⊥ AE and CF ⊥ AE. Prove that:

a. ΔABG ~ ΔDCB

b. $\frac{BC}{BD} = \frac{BE}{BA}$



Solution:



Given: EB ⊥ AC, BG ⊥ AE and CF ⊥ AE.

To prove:

a. Δ ABG ~ ΔDCB

b. $\frac{BC}{BD} = \frac{BE}{BA}$

Proof:

In ΔABG and ΔDCB,

∠2 = ∠5 = 90°

∠6 = ∠4 (corresponding angles)

∴ By AA similarity, ΔABG ~ ΔDCB

∴ ∠1 = ∠3 (Corresponding angles of similar triangles are equal)

In ΔABE and ΔDBC, since

∠1 = ∠3 and

∠ABE = ∠5 = 90°

By AA similarity, ΔABE ~ ΔDBC

$\frac{BC}{BD} = \frac{BE}{BA}$ (Corresponding sides of similar triangles are proportional)