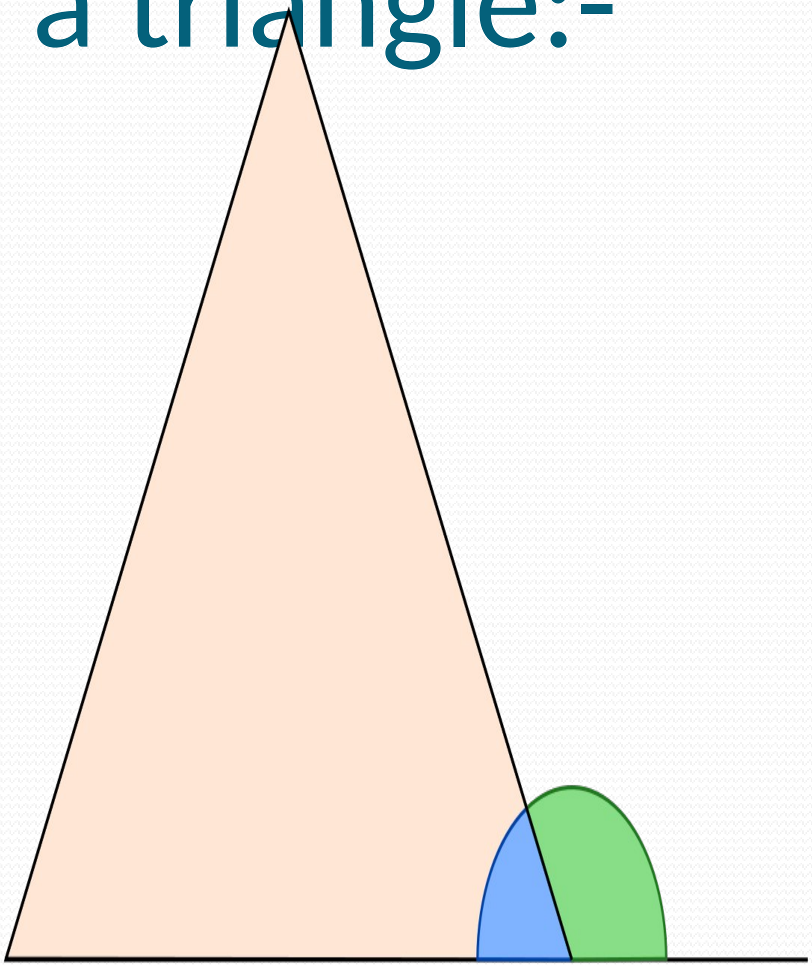
Chapter6- The triangle and its properties- Module 2

Exterior angle property and angle sum property in a triangle

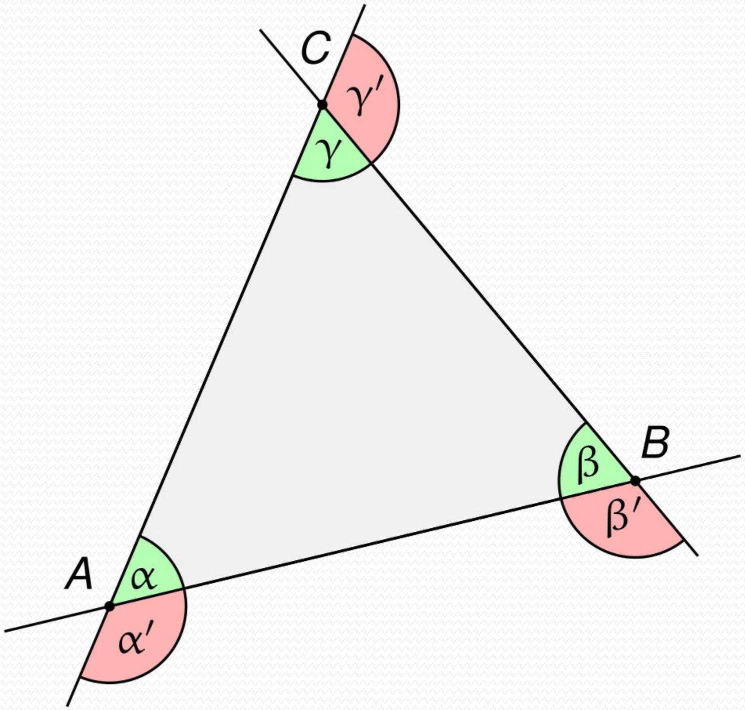
Exterior angle of a triangle:-

- When a side of a triangle is extended in one direction , the angle thus formed is called exterior angle.
- It is adjacent to the interior angle of the triangle at that particular vertex.



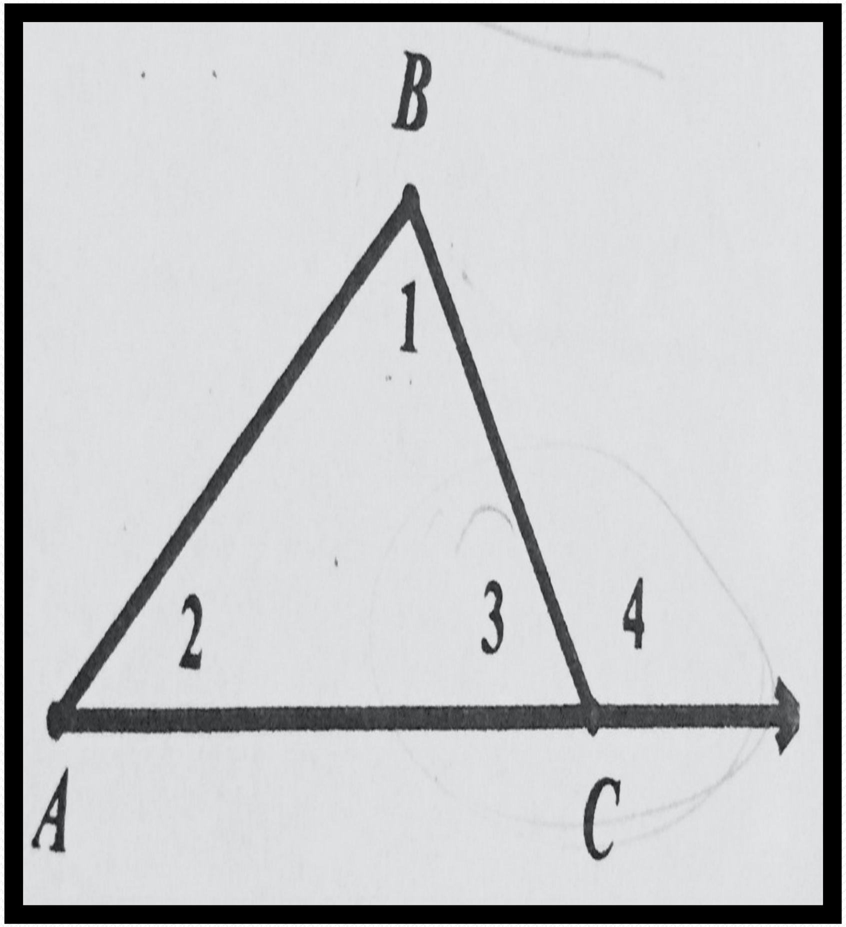
An example -

- In the adjacent figure, for the triangle ABC, α,β and γ are the interior angles at the vertices A,B and C respectively.
- At the same vertices A,B and C the exterior angles are α’,β’ and γ’.
- Here α,β,γ and α’,β’γ’ are the denotions of the angles.(You may use any number/small alphabetical letters in place of these symbols).



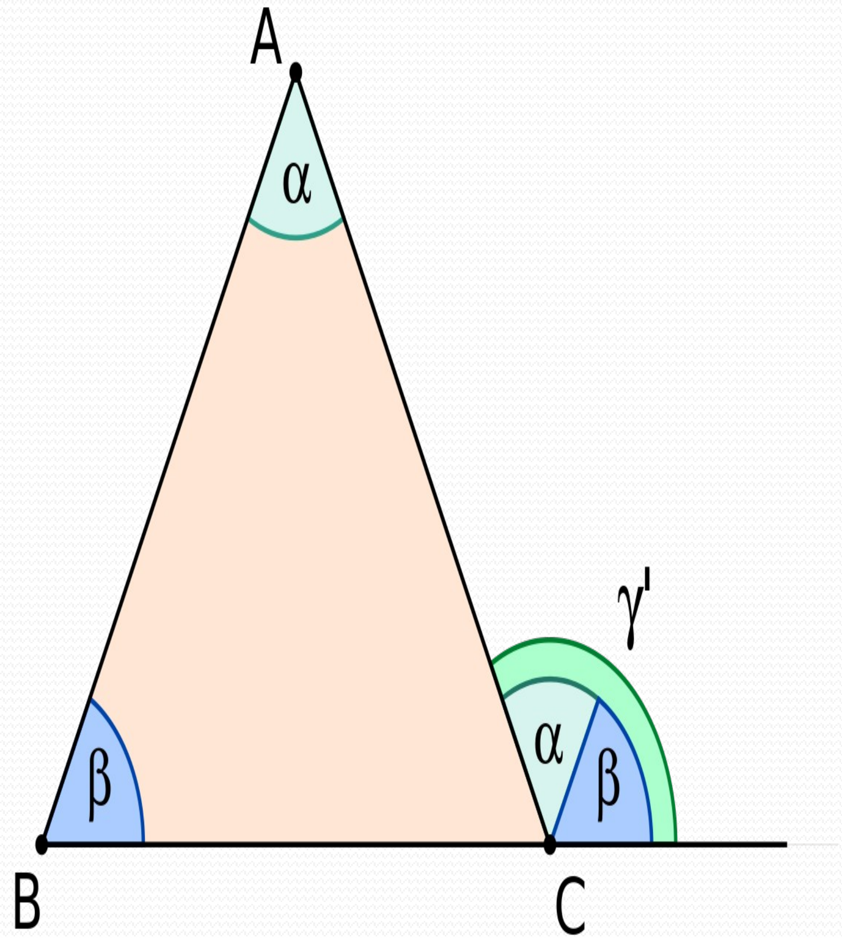
Interior opposite angles

- Draw a triangle ABC and produce one of its side, say AC(as shown in the adjacent figure).
- Observe the exterior angle formed at the point C.
- Here ∠4 is the adjacent angle of ∠3.
- ∠1 and ∠2 are the two interior opposite angles of the exterior angle ∠ 4.



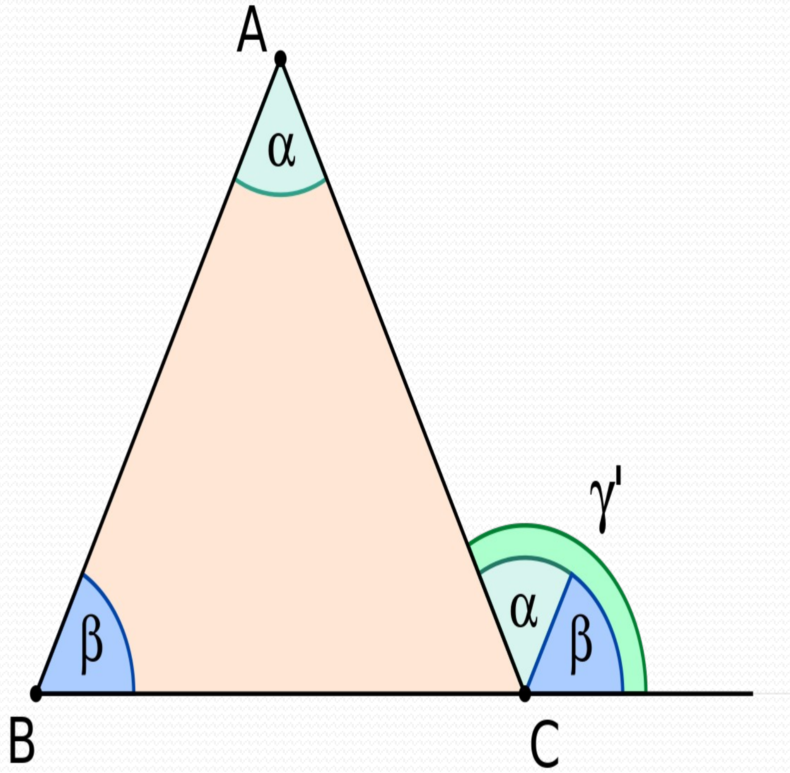
Hands on activity:-

- Drawa a triangle ABC.
- Extend one of the sides (let BC).
- Now take a protractor and measure the exterior angle formed at the point C(Let γ’)
- Now measure the interior angles at the points A and B(let α and β).

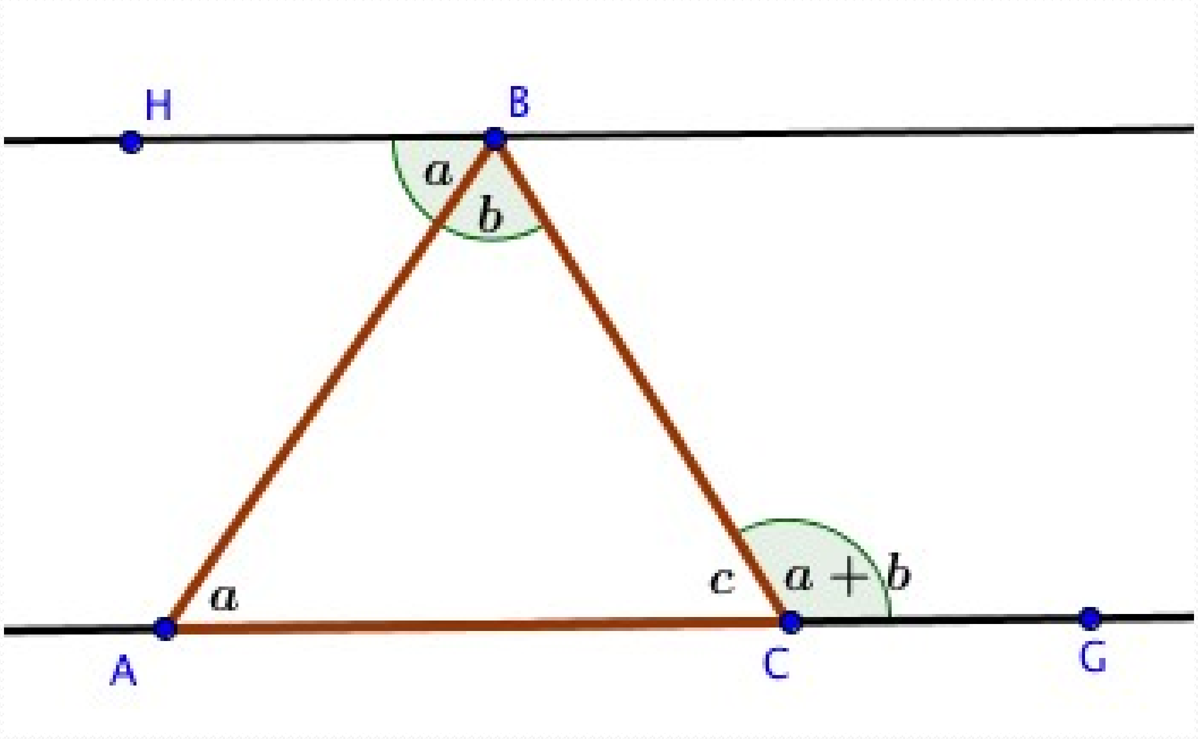


Continued:-

- Find m∠A+ m∠B.
- Compare the sum with the measure of exterior angle at the point C.
- What do you observe?
- Exterior angle is equal to the sum of interior opposite angles.
- $\gamma' = \alpha + \beta$



Geometrical justification:-



Continued:-

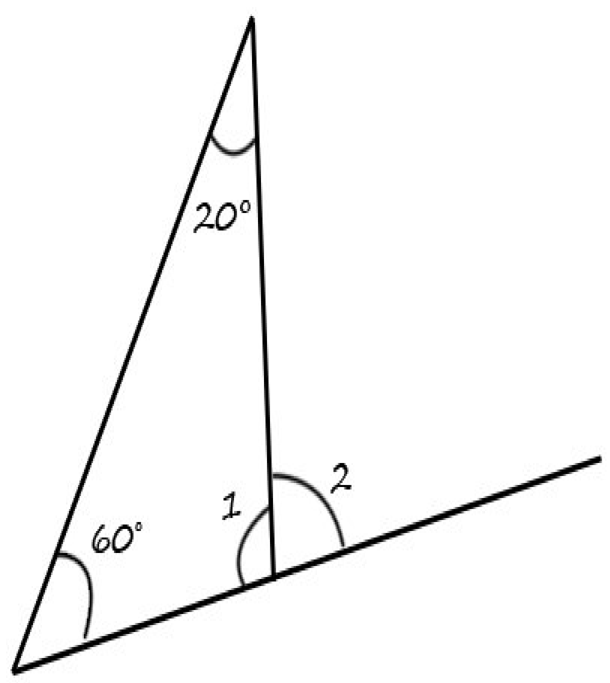
- Let us take a triangle ABC.
- Let the side AC be extended in one direction and passes through G.
- Let us draw a line passing through the vertex B and parallel to the side AC.
- Let the line be BH.
- We can see, m∠HBA=m∠BAC (Since BH is parallel to AC and AB is a transversal line. Therefore, the alternate interior angles are equal.)
- m ∠ HBA = a°,
- m∠HBC = a°+b°(according to figure).

Continued:-

- m∠HBC =m ∠BCG, (since BH is parallel to AG and BC is a transversal line. Therefore alternate interior angles are equal).
- m∠BCG = a°+b°
- m∠BCG = m ∠A + ∠B.
- *Hence , an exterior angle of a triangle is equal to the sum of its interior opposite angles.*

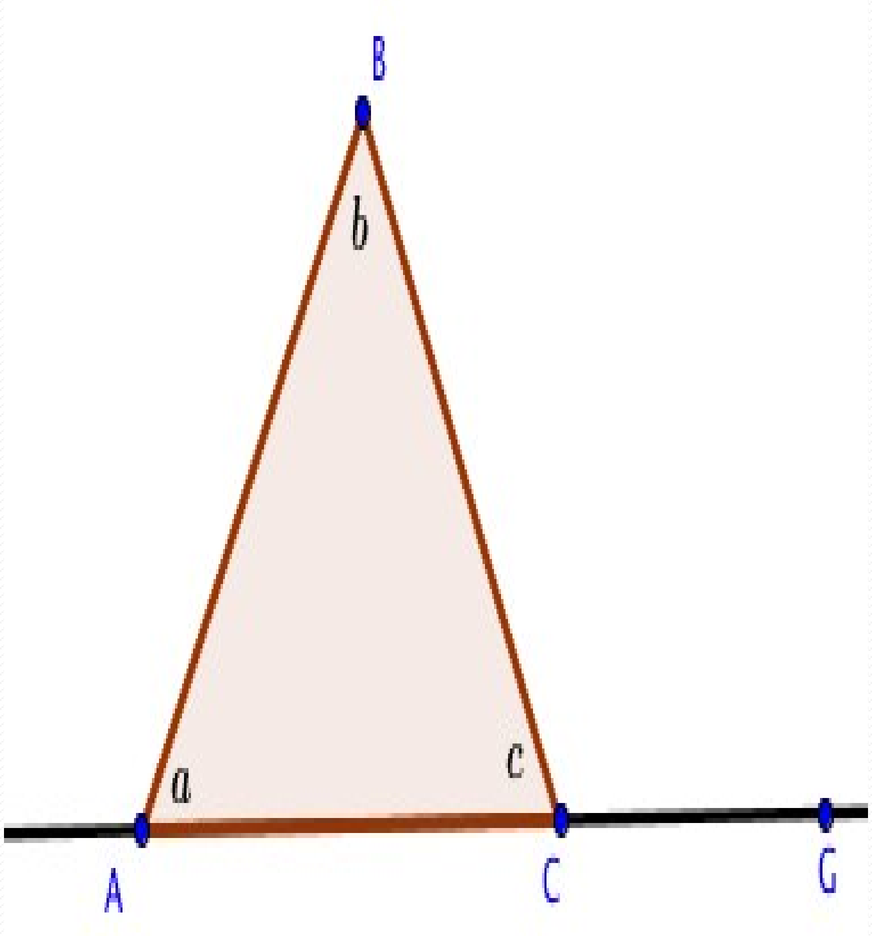
An illustrative example:-

- In the given figure, find the value of ∠1 and ∠2.
- Solution:- Here, ∠2 is an exterior angle of this triangle ,it is equal to the sum of opposite interior angles.
- $\angle 2 = 20° + 60° = 80°$
- Now, ∠1 and ∠2 are linear pair of angles,
- $\angle 1 = 180° – 80° = 100°$.



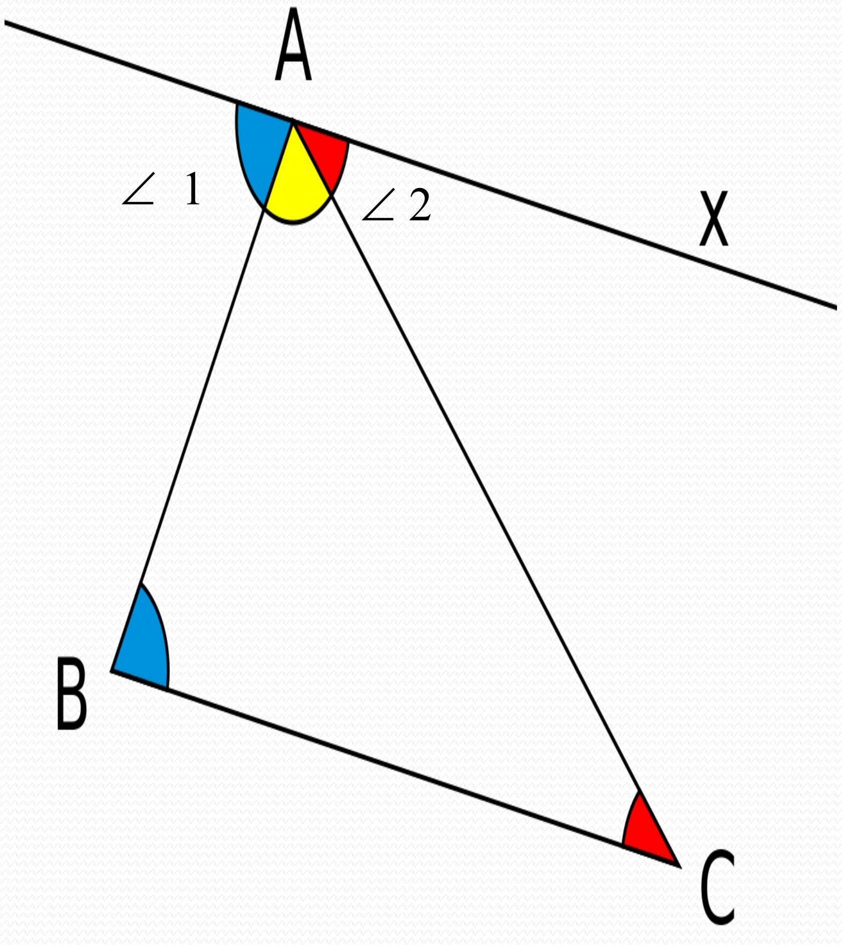
Angle sum property of a triangle

- In a triangle , the sum of all three interior angles is always equal to 180°.
- In this triangle ABC, the sum of all three angles a°+b°+c° =180°.
- In other words, m ∠A+ m∠ B+ m∠ C = 180°.



Hands on activity:-

- Draw any triangle ABC.
- Trace the angle ∠B and ∠C on a tracing paper.
- Cut out the copies of ∠ B and ∠ C from the tracing paper.
- Let us name copies of ∠B = ∠ 1 and ∠C=∠2

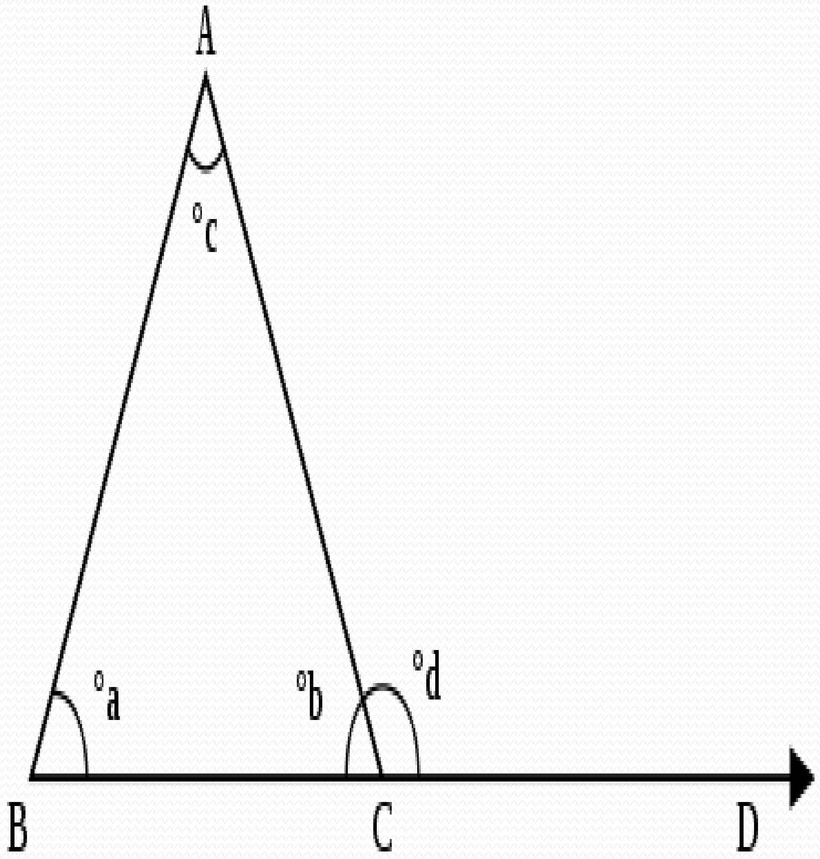


Continued:-

- Put the two pieces adjacent to the vertex A, such that the vertices of two angles($\angle 1$ and $\angle 2$) and the point A coincide.
- What do you observe?
- $\angle 1 + \angle A + \angle 2 = 180°$.(straight angle)
- It means $m\angle A + m\angle B + m\angle C = 180°$.

An illustrative examples

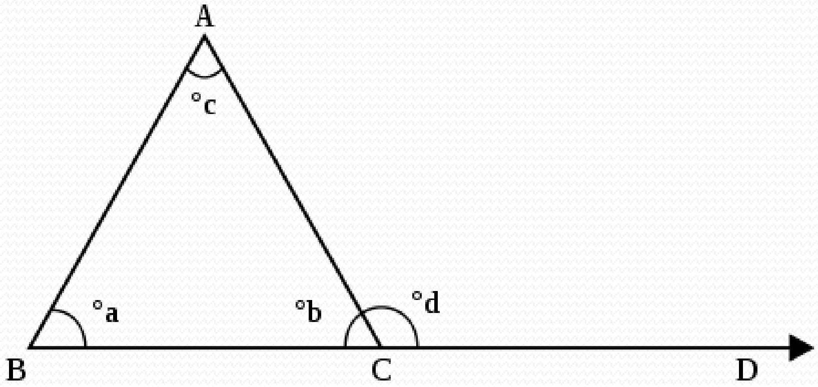
- If a:b:c=2:3:4 . Then find the value of a,b,c and d.
- Solution:- We know that the sum of all the angles of a triangles is 180°.
- Let a= 2x°,b= 3x° and c = 4x°
- Since, a°+b°+c° = 180°



Continued

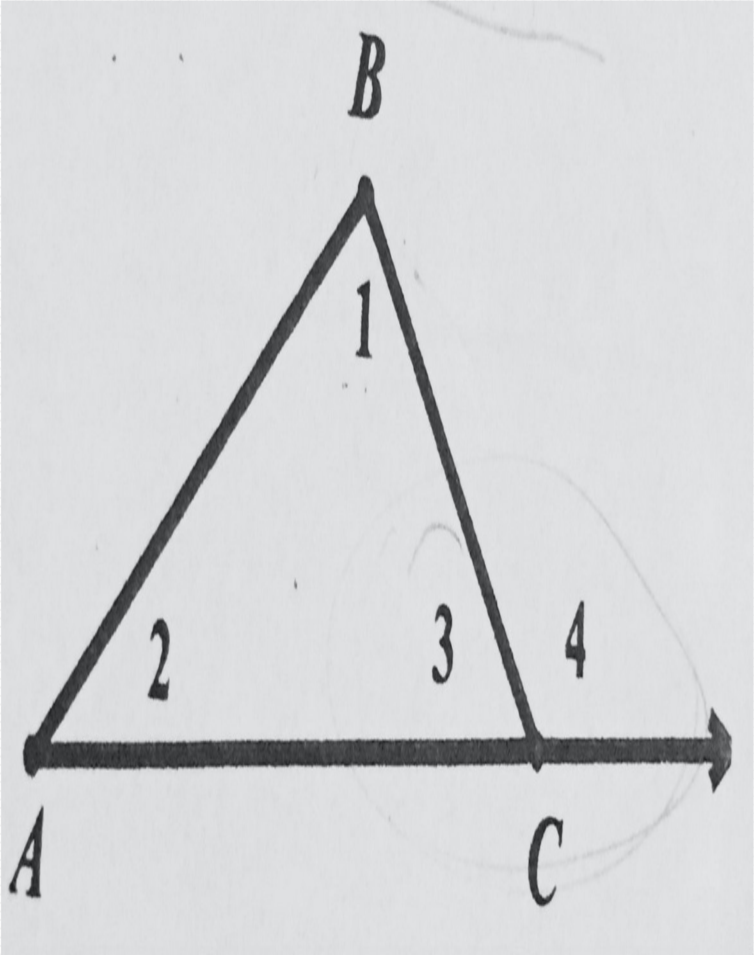
- $2x° + 3x° + 4x° = 180°$
- $9x° = 180°$
- X = 20
- A= 40°,
- B=60°,
- C=80°
- Now, c+d=180°,

D= 180° – 60° = 120°.



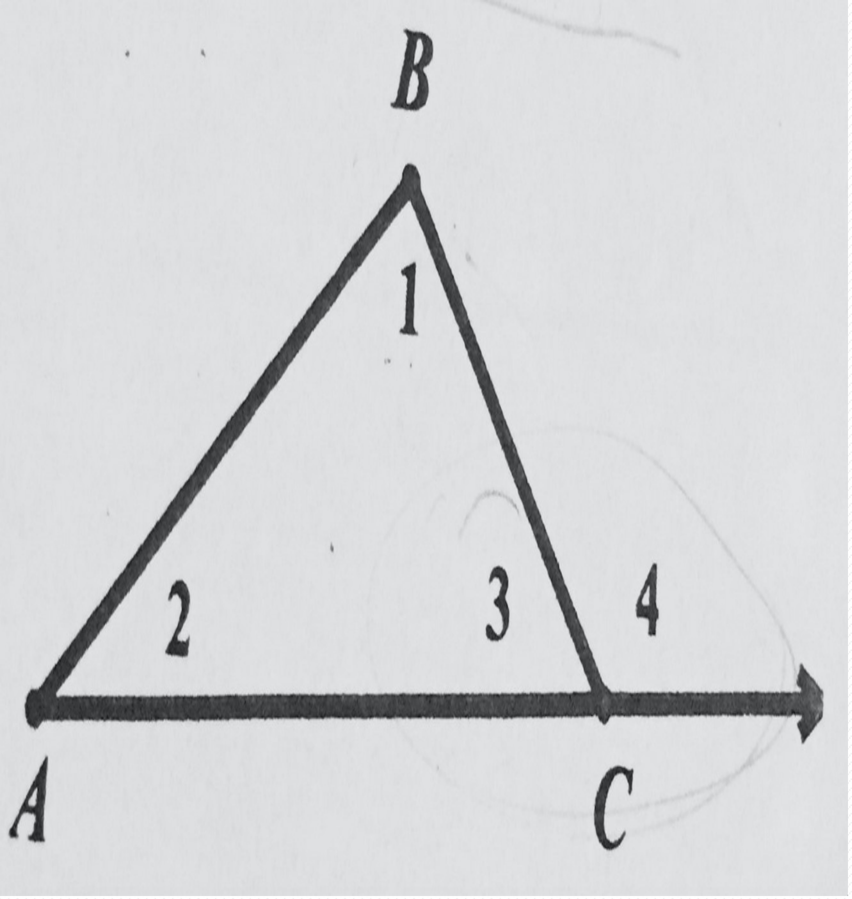
Geometricals Justification:-

- **Statement:- The total measure of the three angles of a triangle is 180°.**
- Given:- ∠ 1,∠2 and ∠3 are the interior angles of the triangle ABC.
- ∠ 4 is the exterior angle when AC is extended in one direction.



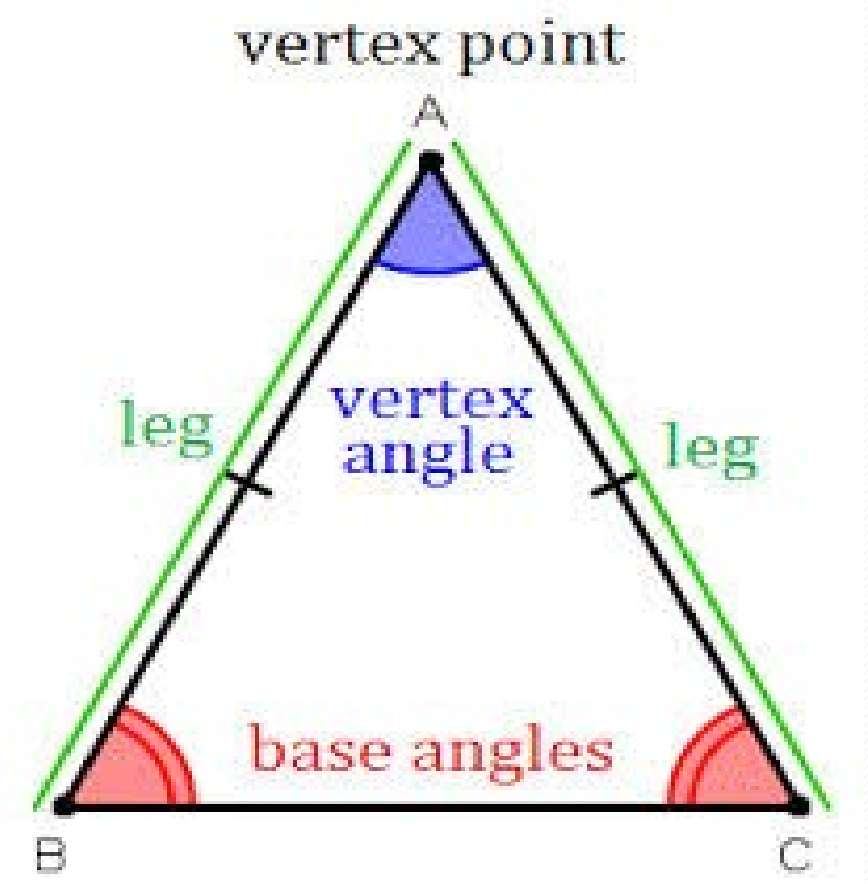
Continued:-

- Justification:- $\angle 1 + \angle 2 = \angle 4$ (by exterior angle property)
- $\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 3$ (adding angle 3 to both the sides).
- But $\angle 4$ and $\angle 3$ form the linear pair so it is 180°.
- ***Therefore , $\angle 1 + \angle 2 + \angle 3 = 180°$.***



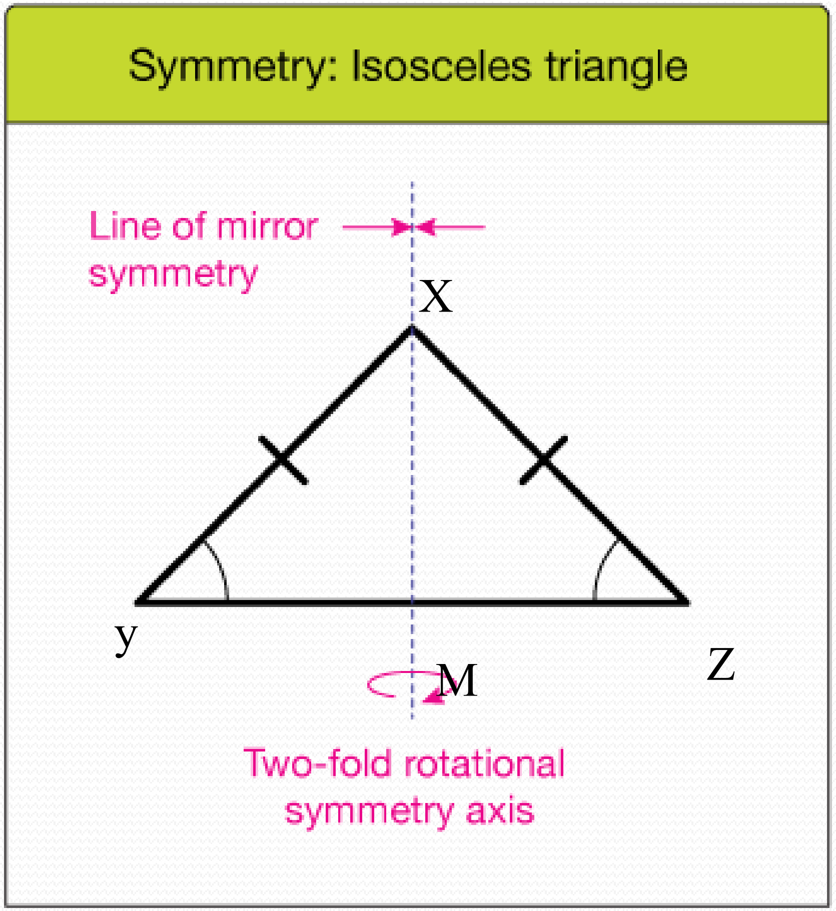
Special type of triangles:-

- Isosceles triangles:-
- It is a special type of triangle in which two sides are equal.
- The angles opposite to equal sides are also equal.
- Two equal angles are called base angles.
- The third angle is called vertex angle.



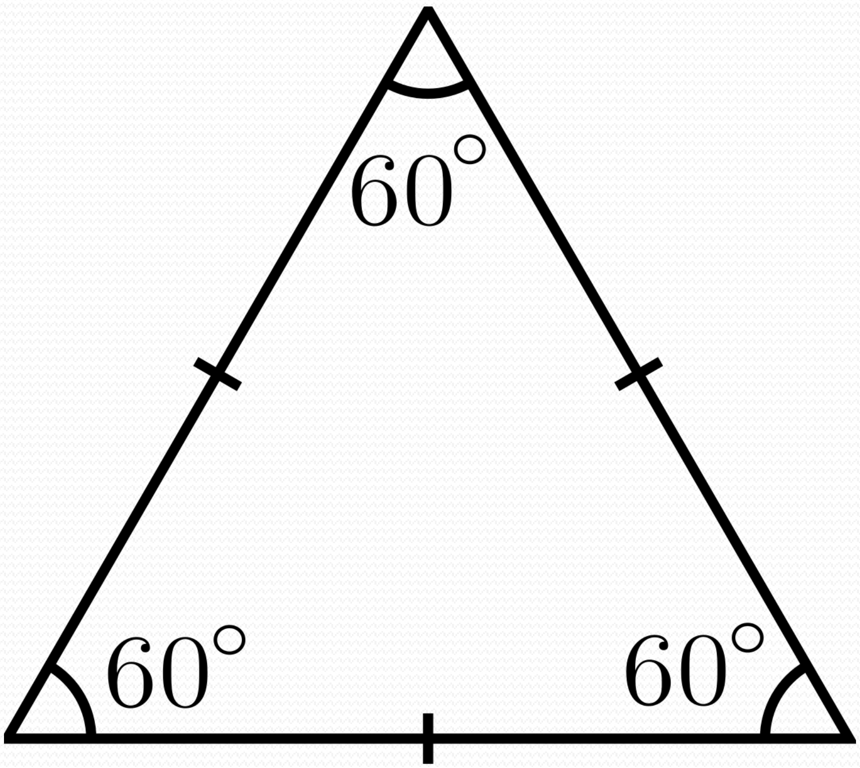
Hands on activity:-

- **Statement:- In an isosceles triangle the angles opposite to equal sides are equal**
- From a piece of paper cut out an isosceles triangle XYZ, with XY=XZ.
- Fold it such that Z lies on Y.
- The line XM through X is now the axis of symmetry.
- If you observe , you can find that $\angle Y$ and $\angle Z$ fit on each other exactly.
- It means that $\angle Y$ and $\angle Z$ are equal to each other.
- **Base angles of an isosceles triangle are always equal.**



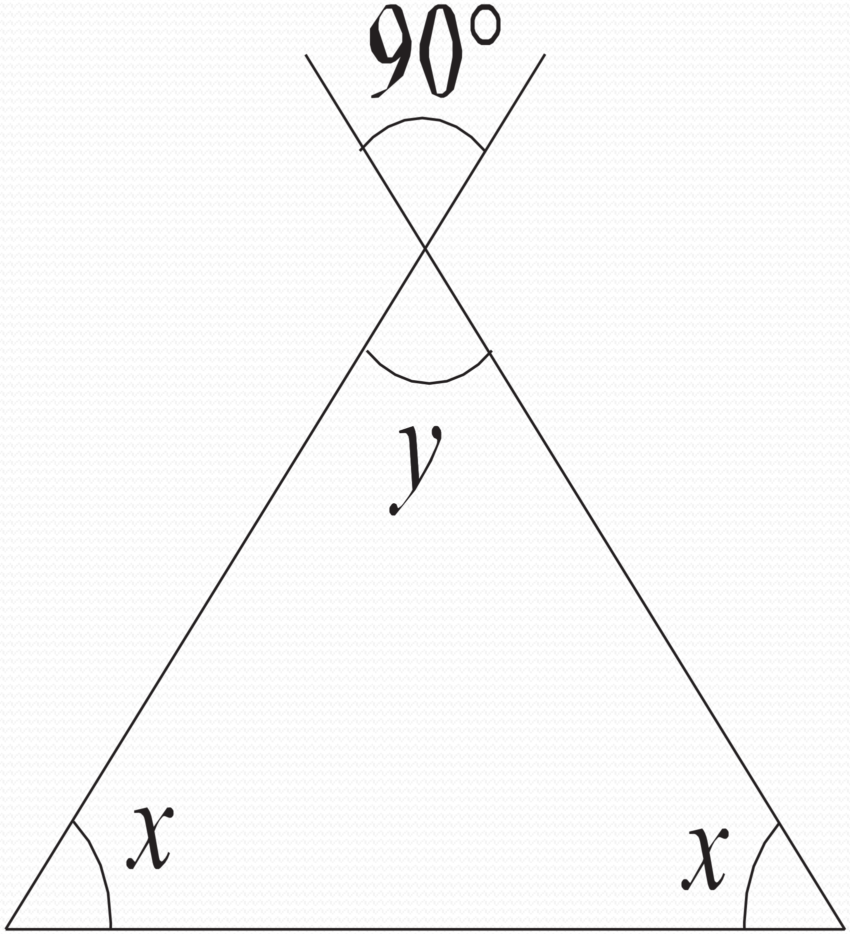
Equilateral triangle:-

- It is a special type of triangle.
- The three sides of this triangle are equal.
- The three angles of this triangle are always equal and equal to 60°.



An illustrative example:-

- Find the value of x and y.
- Solution:- Here, y = 90° (vertically opposite angles are equal)
- Since x+x+y = 180°(angle – sum - property)
- 2x + 90° = 180°
- 2x = 90°
- X= 45°




THANK YOU!